Thermodynamics of strongly gravitating shells and the entropy of non-extremal and extremal black holes

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Black hole entropy, S, is one of the most fascinating issues in contemporary physics, as one does not yet strictly know what are the degrees of freedom at the fundamental microlevel, nor where are they located precisely. In addition, extremal black holes, in contrast to non-extremal ones, present a conundrum, as there are two mutually inconsistent results for the entropy of extremal black holes. There is the usual Bekenstein-Hawking S=A/4 value, where A is the horizon area, obtained from string theory and other methods, and there is the prescription S=0 obtained from the fact that for extremal black holes the period of the Euclidean time is not fixed in a classical calculation of the action. In order to better understand black hole entropy in its generality, and in particular in the extremal limit, we exploit a matter based framework and use a thermodynamic approach for an electrically charged thin shell. We find the entropy function for such a system. We then take the shell radius into its gravitational radius (or horizon) limit. This limit is the quasiblack hole limit. We show that: (i) For a non-extremal shell the gravitational radius limit yields S=A/4. The contribution to the entropy comes from the pressure. (ii) For an extremal shell the calculations are very subtle and interesting. The horizon limit gives an entropy which is a function of the horizon radius alone, but the precise functional form depends on how we set the initial shell. The values 0 and A/4 are certainly possible values for the extremal black hole entropy. This formalism clearly shows that non-extremal and extremal black holes are different objects. In addition, the formalism suggests that for non-extremal black holes all possible degrees of freedom are excited, whereas in extremal black holes, in general, only a fraction of those degrees of freedom manifest themselves. We conjecture that for extremal black holes the entropy S is restricted to the interval between 0 and A/4. Since an extremal shell has zero pressure, the contribution to the entropy comes from the shell's electricity. (iii) There is yet another possibility: to take the extremal limit concomitantly with the gravitational radius limit. In this case, and contrary to the two previous cases, remarkably, both the pressure and the electricity on the shell contribute to the entropy to give S=A/4.