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Regular black holes in Einstein-Maxwell theory

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Outline

- Motivation: black hole interiors
- Introduction:
  – Black holes and singularities
  – First regular black hole solutions
- Charged fluids of Weyl type
- The Guilfoyle solutions
- Regular charged black holes in Guilfoyle solutions
- Summary/Conclusion

Introduction

Black holes and singularities in GRT

Vacuum solutions

- Schwarzschild black hole (m): an event horizon at $r_h = 2m$, $G = 1 = c$, shielding a singularity at $r = 0$, with asymptotically flat regions, formed from the collapse of a nonrotating matter (a star).

- Reissner-Nordström black hole (m, q): an event horizon at $r_h = r_+$, Cauchy horizon at r_-, $r_\pm = m \pm \sqrt{m^2 - q^2}$, a singularity inside r_-, formed by collapse of nonrotating charged matter $(m^2 > q^2)$.

- Kerr (m, J), and the Kerr-Newman (m, q, J) solutions: an event horizon at $r_h = r_+$, Cauchy horizon at r_-, $r_\pm = m \pm \sqrt{m^2 - q^2 - J^2/m^2}$, a ring singularity inside r_-, formed by collapse of rotating charged matter $(m^2 > a^2 + q^2)$.

Refs: Misner, Thorne & Wheeler book 1973; Griffiths & Podolsky book 2009.

Introduction

Black holes and singularities in GRT

Vacuum solutions and singularities

- Schwarzschild black hole : Kretschmann scalar $K = \frac{46m^2}{r^6} \Longrightarrow$ singularity at $r = 0$.

- Reissner-Nordström black hole : $K = \frac{46}{r^6}\left[\left(m - \frac{q^2}{r}\right)^2 + \frac{q^4}{6r^2}\right] \Longrightarrow$ singularity at $r = 0$.

- Kerr and the Kerr-Newman solutions: Ring singularity at $r^2 + J^2 \cos^2\theta / m^2 = 0$.

– The singularity theorems (Penrose 1965, 1978), + physical conditions, prove that singularities are inevitable.

– Those physical (energy) conditions may not hold (due to quantum effects) at very high energy densities, when a singularity would be about to form.

Introduction

Avoiding singularities in GRT

– Cosmological singularities could be avoided by matter with a de Sitter equation of state $p = -\rho_m$ (Sakharov 1966, Gliner 1966, 1975).

$p = -\rho_m$ is equivalent to a cosmological constant (false quantum vaccum term) $T_{\mu\nu} = \Lambda g_{\mu\nu}$ (Zel'dovich 1968).

– An unlimited increase of spacetime curvature during a collapse process could be halted if quantum fluctuations dominate the process $\Longrightarrow$ upper bound to the value of the curvature $\Longrightarrow$ formation of a central core.

– **The Bardeen regular black hole** (1968): The first regular black hole - modification of the Reissner-Nordström metric (with a magnetic type of matter) which near the center tend to a de Sitter solution.

– Most of the subsequent regular black hole solutions are based on Bardeen's proposal. There has been a tremendous development on the implementation and on the analysis of the properties of regular black hole solutions (see e.g. Ansoldi 2008, arXiv:0802.0330, for a review).

Introduction

Regular black hole solutions

· A useful way to classify the regular black hole solutions is:

1. No junction: solution is continuous throughout spacetime.
2. Two simple regions: solution has boundary surfaces joining the two regions.
3. Two regions: the solution has a surface layer, i.e., thin shell, joining the two regions.

Introduction

1. No junction

- Based on Gliner (1966, 1975) proposal, Dymnikova (GRG 1992) proposed a black hole model in which the core is de Sitter and gives way in a smooth manner into a Schwarzschild solution, with Cauchy and event horizons. Several subsequent works developing this idea followed (Dymnikova 1996, 2000, 2001, 2003, 2004, 2005, 2010, Gliner 1998).
- Ayón-Beato and Garcia (PLB 2000, GRG 2005) invoked nonlinear fields and sources to generate the Bardeen model as a nonlinear magnetic monopole (see Bronnikov (PRL 2000, PRD2001), Matyjasek (PRD2004) found the extremal limit).
- Bronnikov and collaborators (Fabris, Dehnen, Melnikov, Dymnikova, PRL 2006, GRG 2007, CQG 2007): several regular black holes in which the source are fields permeating the whole spacetime, the core is an expanding universe with de Sitter asymptotics and the outer region tends to Schwarzschild. See also Matyjasek, Tryniecki, and Klimek (MPLA 2008).

Introduction

2. Solutions with boundary surfaces

- Can construct regular black holes by filling the inner space with matter up to a certain surface and make a smooth junction, through a boundary surface, to the Schwarzschild solution (Mars CQG 1996, Magli RMP 1999, Elizalde and Hildebrandt PRD 2002, Conboy and Lake PRD 2005). The junction to Schwarzschild is made through a spacelike surface, rather than an usual timelike surface.
- Regular Schwarzschild black holes in which the boundary surface is lightlike or timelike have not been found in the literature.

3. Solutions with boundary surfaces

Solutions with boundary layers, i.e., thin shells It is possible and of interest to make the transition from an inner de Sitter core to an outer Schwarzschild, Reissner-Nordström, or other spacetime, through surface layers, or thin shells. Regular black holes with thin shells of **spacelike**, **lightlike**, and **timelike** character have been found.

a. Spacelike thin shells
Following Zel'dovich's idea (1968), Markov (AP 1984) suggested an upper bound for the curvature, of the order of the Planck curvature. After it is achieved the matter turns into a de Sitter phase. The transition is made through a spacelike thin shell. It was developed by Lake and Zannias (PLA 1989), Frolov, Markov, and Mukhanov (PRD 1990), Balbinot and Poisson (PRD 1990), Balbinot (PRD 1990), Morgan (PRD 1991), Barrabes and Frolov (PRD "How many new worlds are inside a black hole?" 1996).
See also Burinskii, Elizalde, Hildebrandt, and Magli (PRD 2002) for a general discussion including the Kerr-Newman metric.

3. Solutions with boundary surfaces

b. Lightlike thin shells

– Gonzalez-Diaz (LNC 1981) tried a solution by direct matching of de Sitter spacetime with the Schwarzschild solution on the horizon, a null surface.

– Shen and Zhu (PLA 1989) reanalyzed later this soldering of de Sitter spacetime with the Schwarzschild solution, while Shen and Tan in 1989 (PLA 1989) generalized the Gonzalez-Diaz idea to d dimensions.

– Daghigh, Kapusta, Hosotani (Arxiv 2000): Schwarzschild type matching can also be achieved within a more general parametrization of the static metric by two different functions due to the jump of the product $g_{tt}g_{rr}$.

– However, Gron and Soleng (LNC 1985, PLA 1989) showed that the direct matching onto Schwarzschild at the horizon is incorrect.

– Poisson and Israel (CQG 1988): de Sitter spacetime cannot be soldered directly to an exterior Schwarzschild vacuum at the horizon, since the junction conditions would be violated. It is necessary to put a thin shell of noninflationary material at a junction outside the event horizon.

Introduction

3. Solutions with boundary surfaces

b. Lightlike thin shells

– Gal'tsov and Lemos (CQG 2001): a no-go theorem showing that the more general matching proposed in Daghigh, Kapusta, Hosotani (Arxiv 2000) is also not possible (see also Bronnikov PRD 2001).

– Extending to the Reissner-Nordström spacetime, Shen and Zhu (GRG 1985, NC 1985) tried the same type of matching. By including charge the matching problems of the Schwarzschild case may be avoided.

– Barrabès and Israel (PRD 1991) gave interesting examples of a lightlike thin shell matching at the Cauchy horizon (see also Barrabès and Hogan book 2003 for null matching).

Introduction

3. Solutions with boundary surfaces

c. Timelike thin shells

– For regular uncharged black holes with boundary layers or thin shells, timelike matching is not found in the literature.

– Regular charged black holes with a de Sitter core and a thin shell of electric charge with a timelike matching has been found recently (Lemos and Zanchin, PRD 2011).

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Regular black holes either with a charged (usually magnetic) core, or with a de Sitter core (or, more generally, $p < -\rho$) at the center are known, but with electric charge and matter with $p < -\rho$ together seem not to have been explored. So, to study such cases is also a local motivation within the larger context.

Regular black holes with electric charge

A de Sitter core with a charged thin-shell - Lemos, VTZ PRD 2011

- The idea: a de Sitter core, an electric coat (thin-shell), a Reissner-Nordström spacetime outside.
- Implies: if there are horizons, the matter is inside the Cauchy horizon, boundary is timelike (like in a star), except in one limiting case, where it is lightlike.

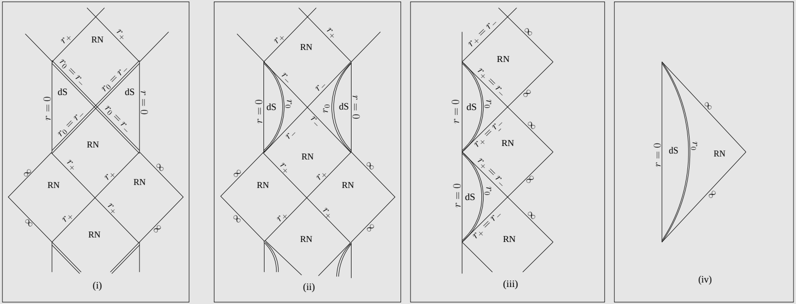

Figure: Carter-Penrose diagrams

Regular black hole solutions

Regular black holes: some important facts

- Regular black holes require “exotic” matter.
- (i) de Sitter: $p = -\rho_m$ (Gliner & Dymnikova, 1975; Dymnikova, 1992, 1996; ...)
- (ii) Nonlinear electromagnetic fields (Bardeen, 1968; Ayón-Beato & García, 2000, 2005; Bronnikov, 2000, 2001)
- (iii) Phantom matter: $p + \rho_m < 0$ (Bronnikov & Fabris, 2005; Bronnikov & Dymnikova, 2007; Fabris et al., 2009, 2014; ...)
- (iv) Charged matter with a de Sitter core (Shen & Zhu, 1985; Lemos & VTZ, 2011; Azreg-Aïnou, 2014)
- (v) Charged phantom matter: $p + \rho_m < 0$ (Guilfoyle solutions, Lemos & VTZ 2015 - in the following)

Charged fluids of Weyl type

Basic equations - d-dimensions

Einstein-Maxwell equations ($c = 1$):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{d-2}{d-3} S_{d-2} G_d \left(T_{\mu\nu} + E_{\mu\nu} \right) ,$$
$$\nabla_\nu F^{\mu\nu} = S_{d-2} J^\mu ;$$

$R_{\mu\nu}$ = Ricci tensor,
$T_{\mu\nu} = (\rho_m + p) U_\mu U_\nu + p g_{\mu\nu}$,
$S_{d-2} E_{\mu\nu} = F_\mu{}^\rho F_{\nu\rho} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}$,
$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\mu A_\mu$,
U_μ = four-velocity
A_μ= gauge potential,
$J_\mu = \rho_e U_\mu$ = current density,
∇_μ = covariant derivative.

Charged fluids of Weyl type: analysis

Basic equations

Static d-dimensional spacetime:

$$ds^2 = -W^2 dt^2 + h_{ij} dx^i dx^j \,, \qquad U_\mu = W\,\delta^0_\mu \,, \qquad A_\mu = -\phi\,\delta^0_\mu \,,$$

W, h_{ij} e ϕ = function of the spatial coordinates alone.

Einstein-Maxwell equations yield

$$\nabla^2 W = \frac{G}{W}\,(\nabla_i\phi)^2 + S_{d-2}\,G\,W\left(\rho_m + \frac{d-1}{d-3}\,p\right),$$

$$\nabla^2\phi = -\frac{\nabla_i W}{W}\nabla^i\phi - S_{d-2} W\,\rho_e,$$

Conservation Eq., $\nabla_\mu E^{\mu\nu} + \nabla_\mu T^{\mu\nu} = 0$:

$$(\rho_{\rm m} + p)\,\frac{\nabla_i\, W}{W} + \rho_{\rm e}\,\frac{\nabla_i\,\phi}{W} + \nabla_i\, p = 0.$$

Charged fluids of Weyl type: nonzero pressure

If (Weyl Ansatz)
$W = W(\phi) \Longrightarrow [(\rho_{\rm m} + p)\, W' + \rho_{\rm e}]\, \frac{\nabla_i \phi}{W} + \nabla_i p = 0 \Longrightarrow p = p(\phi)$, but no single relation $W(\phi)$.

Simple assumptions:

1- Weyl (1917) relation $W^2 = \left(-\varepsilon\sqrt{G_{\rm d}}\phi + b\right)^2 + c \;\; (\varepsilon = \pm 1) \;\; \Longrightarrow$

$$\rho_{\rm e} = \varepsilon\sqrt{G_{\rm d}}\left(\rho_{\rm m} + \frac{{\rm d}-1}{{\rm d}-3}p\right)$$ (Lemos & VTZ, 2005, 2009; Gautreau & Hoffman, 1973, for d=4).

- $\Longrightarrow p \sim W^{2/(d-3)}$ (Lemos & VTZ, 2005; Guilfoyle 1999 (d=4)).
- $\Longrightarrow$ No compact objects: $p = 0$ implies $W = 0$ (infinite redshift).

Charged fluids of Weyl type: $p \neq 0$

2- If (Guilfoyle ansatz) $W^2 = a\left(-\varepsilon\sqrt{G_{\rm d}}\phi + b\right)^2 + c$, the charged pressure fluid quantities satisfy the constraint (L & Z, 2009)

$$a\,\rho_{\rm e}\left(-\varepsilon\sqrt{G_{\rm d}}\phi + b\right) = \varepsilon\sqrt{G_{\rm d}}\left(\rho_{\rm m} + \frac{{\rm d}-1}{{\rm d}-3}p + \varepsilon\,(1-a)\,\rho_{\rm em}\right)W\,,$$

$$a\,\rho_{\rm e} = \varepsilon\sqrt{G_{\rm d}}\left(\rho_{\rm m} + \frac{{\rm d}-1}{{\rm d}-3}p + \varepsilon\,(1-a)\,\rho_{\rm em}\right)\,, \quad \text{if } c = 0,$$

where $\left(\rho_{\rm em} = \frac{1}{S_{d-2}}\frac{(\nabla_i\phi)^2}{W^2}\right)$. The above relations can be considered as equations of state satisfied by the charged fluid.

- $a > 1 \Longrightarrow$ positive pressure, undercharged objects ($m^2 > q^2$).
- $a = 1 \Longrightarrow$ zero pressure, extremely charged objects ($m^2 = q^2$).
- $0 < a < 1 \Longrightarrow$ negative pressure, overcharged objects.
- Guilfoyle ansatz is equivalent to take $dp/d\phi$ as a particular combination of ρ_m, ρ_e and ρ_{em}.

Guilfoyle solutions

$$ds^2 = -B(r)\,dt^2 + A(r)\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right) \tag{1}$$

$W^2 \equiv B(r) = a(\phi(r)+b)^2 + c$ (b can be put to zero) (Guilfoyle relation)

$$8\pi\rho_{\mathrm{m}}(r) + \frac{Q^2(r)}{r^4} = \frac{3}{R^2} = \text{const.} \text{ (Cooperstock-de La Cruz; Florides)}, \tag{2}$$

$$A(r) = \left(1 - \frac{r^2}{R^2}\right)^{-1}, \tag{3}$$

$$Q(r) = 4\pi\int_0^r \rho_{\mathrm{e}}(r)\frac{r^2 dr}{\sqrt{1-\frac{r^2}{R^2}}} = \frac{r^2}{\sqrt{B(r)}}\sqrt{1-\frac{r^2}{R^2}}\,\frac{d\phi(r)}{dr}, \tag{4}$$

Boundary surface $r = r_0$ (implying $p(r_0) = 0$)

$$\Rightarrow \frac{1}{R^2} = \frac{1}{r_0^3}\left(2m - \frac{q^2}{r_0}\right). \tag{5}$$

Guilfoyle solutions: Class Ia ($c = 0$) solutions:

$$B(r) = \left[\frac{2-a}{a^{1+1/a}}\,F(r)\right]^{2a/(a-2)}, \quad F(r) = k_0 R^2\sqrt{1-\frac{r^2}{R^2}} - k_1\,, \tag{6}$$

$$8\pi\rho_{\rm m}(r) = \frac{3}{R^2} - \frac{a}{(2-a)^2}\,\frac{k_0^2\,r^2}{F^2(r)}\,, \tag{7}$$

$$Q(r) = \frac{\varepsilon\sqrt{a}}{2-a}\,\frac{k_0\,r^3}{F(r)}, \tag{8}$$

$$8\pi p(r) = -\frac{1}{R^2} + \frac{a}{(2-a)^2}\,\frac{k_0^2\,r^2}{F^2(r)} + \frac{2k_0\,a}{2-a}\,\frac{\sqrt{1-r^2/R^2}}{F(r)}\,, \tag{9}$$

$$k_0 = \frac{|q|\,a^{2/a}}{r_0^3}\left(\frac{m}{q} - \frac{q}{r_0}\right)^{1-2/a}, \tag{10}$$

$$k_1 = \sqrt{1-\frac{r_0^2}{R^2}}\left[k_0\,R^2 - \frac{a^{1+1/a}}{2-a}\left(1-\frac{r_0^2}{R^2}\right)^{-1/a}\right]. \tag{11}$$

Guilfoyle solutions: Class Ia solutions

Parameters and relations

- Five parameters: a, R, r_0, m, q.
- Two relations:

$$m = \frac{r_0}{2}\left(\frac{r_0^2}{R^2} + \frac{q^2}{r_0^2}\right). \tag{12}$$

$$a = \frac{r_0^2}{4q^2}\left(\frac{r_0^2}{R^2} - \frac{q^2}{r_0^2}\right)^2 \left(1 - \frac{r_0^2}{R^2}\right)^{-1}. \tag{13}$$

- Stars satisfying energy conditions (Guilfoyle, CQG 1999).
- Quasiblack hole with pressure (Lemos & VTZ, PRD 2010).
- Regular black holes with a de Sitter core (Lemos & VTZ, PRD 2011).
- Extremely compact stars: Buchdahl-Andréasson limit (Lemos & VTZ, CQG 2015).
- Regular black holes with a core of phantom charged matter (Lemos & VTZ, TBP 2015).

Guilfoyle solutions: Regular black holes with charged phantom matter core

Basic conditions

– The exterior metric is the Reissner-Nordström, with event and (maybe) Cauchy horizons:

$$r_h = r_+ = m + \sqrt{m^2 - q^2},$$
$$r_c = r_- = m - \sqrt{m^2 - q^2}$$

$\Longrightarrow m^2 \geq q^2.$

– The matching surface must be inside the event horizon. So we can show that (from the boundary conditions)

$$0 < r_0 \leq r_-.$$

$\Longrightarrow$ Different class of solutions depending on the values of r_0/q and R/q.

Regular black holes with phantom charged matter core

Radii

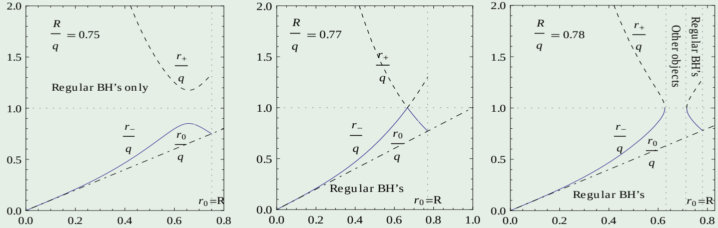

Figure: Large and intermediate densities

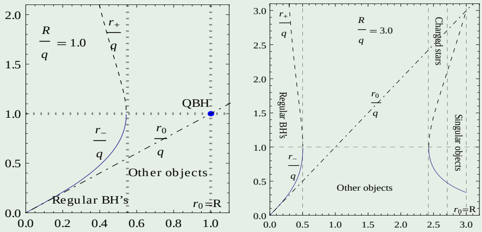

Figure: Small densities

Regular black holes with a phantom charged matter core

$8\pi\rho_{\mathrm{m}}(r) + \frac{Q^2(r)}{r^4} = \frac{3}{R^2}$; *needs small R (large densities)*

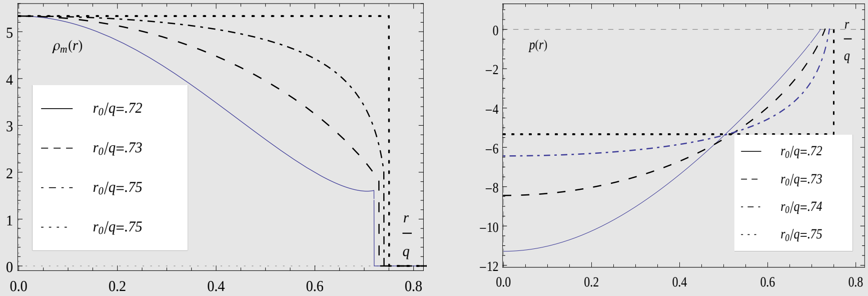

Figure: A plot of the energy density $\rho_{\mathrm{m}}(r)$ and pressure $p(r)$ for $R/q = .75$ and four different $\alpha \equiv r_0/q$: $\alpha = .72$ (solid line), $\alpha = .73$ (dashed line), $\alpha = .74$ (dash-dotted line), and $\alpha = .75$ (dotted line)

Regular black holes with phantom charged matter

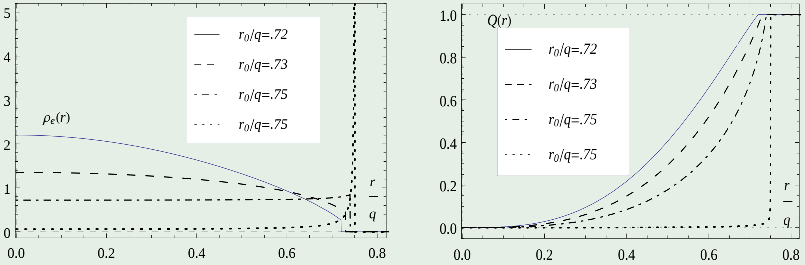

Figure: A plot of the charge density $\rho_e(r)$ and charge function $Q(r)$ for $R/q = .75$ and four different $\alpha \equiv r_0/q$: $\alpha = .72$ (solid line), $\alpha = .73$ (dashed line), $\alpha = .74$ (dash-dotted line), and $\alpha = .75$ (dotted line

Regular black holes with phantom charged matter

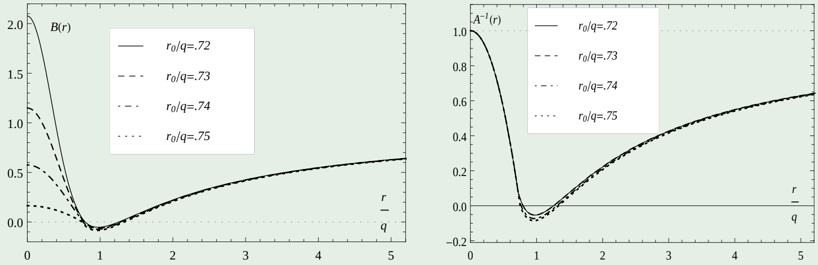

Figure: Graphs of $B(r)$ and $1/A(r)$ for $R/q = .75$ and four different $\alpha \equiv r_0/q$: $\alpha = .72$ (solid line), $\alpha = .73$ (dashed line), $\alpha = .74$ (dash-dotted line), and $\alpha = .75$ (dotted line - corresponding to a regular black hole with timelike boundary $r_0 = r_-$).

Regular black holes with phantom charged matter core

Carter-Penrose diagrams

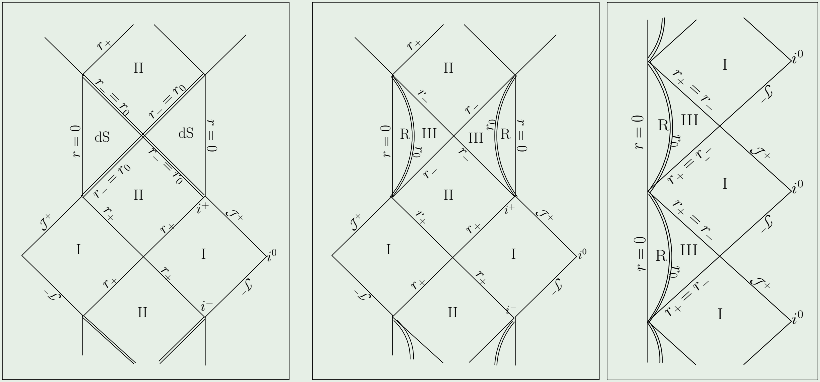

Figure: Left: Lightlike boundary. Center: Lightlike boundary Right:Extreme case, timelike boundary.

The Guilfoyle solutions: parameter space

Work in progress

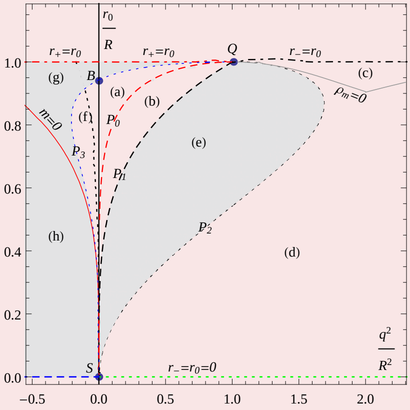

Figure: Grey regions contain singular solutions.

Summary/Conclusion

– A brief historical review of regular black hole solutions was given

– Investigated the Guilfoyle solutions showing regular black holes - with a core of charged phantom matter.

– A further step towards the interpretation of the full spectrum of Guilfoyle solutions.

– Some progress on understanding the black hole interior has been made.

Thank You!