Recent Advances in Engineering Science as Applied to Aeolian Vibration: an Alternative Approach

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SUMMARY

This paper summarizes modern advances of nonlinear mechanics, fluid dynamics, and vortex-induced vibrations as applied to aeolian vibrations. It presents a new and simple method of calculating the aeolian response of a single conductor, which circumvents the uncertainty of the energy input due to wind

1 INTRODUCTION

The name given to aeolian vibration nowadays dates back to the Aeolian harp of the ancient Greeks. The first experimental observations are due to Strouhal [1], who, in 1878, found that the aeolian tones generated by a taut wire in an airstream were dependent only on the airstream velocity and the wire diameter and that the sound was greatly reinforced when the aeolian tones coincided with one of the natural frequencies of the wire. Soon after Strouhal’s paper was published, Lord Rayleigh [2, 3] performed similar experiments and discovered that a violin string vibrating in the wind primarily oscillated in the plane perpendicular to the wind, thus indicating that the oscillating force component normal to the flow was far greater than that parallel to the flow. In 1908, Bénard [4] correlated the musical note studies by Strouhal with two nearly parallel rows of nearly equal-spaced vortices behind a cylinder. In 1911, von Kármán gave his famous theory for the vortex street [5], which stimulated widespread and lasting research on the subject. This then was the beginning for understanding the Kármán vortex street responsible for aeolian vibrations. Up to the middle 1950s, the problem of aeolian oscillations was treated in textbooks on mechanical vibrations and aeroelasticity [6, 7] as one of a mechanical system (usually having one degree of freedom) being excited into vibration by a fluctuating lift force. When the natural frequency of the cylinder coincides with that of the exciting Strouhal frequency, resonance occurs. The cause of the excitation is explained in terms of vortex shedding and the Kármán vortex streets. Aeolian vibration of transmission lines has been known for a long time, at least as far back as the 1920s when the Stockbridge ‘damper’ (dynamic vibration absorber to be exact) was invented. Much has been written on this subject. We cite two recent authoritative publications which are supposed to present a state-of-the-art study of the subject matter. One is a 1978 CIGRE paper [8], and the other is a 1979 reference book published by EPRI [9]. However, they do not cover recent advances. The purpose of this paper is twofold: to try to review modern developments in engineering science as applied to the aeolian vibration of overhead lines in a condensed but readable form, and to present a new method, due to Iwan [10] of calculating the aeolian response of a single conductor in the Reynolds number range of $2 \times 10^2 - 2 \times 10^5$. The pros and cons of the new versus old methods will be elaborated later. Since the recent advances cover several disciplines, i.e., nonlinear mechanics, fluid mechanics and aeroelasticity, a reasonable mathematical exposition starting from first principles will certainly be beyond the scope of this paper. Instead, a fast guided tour is arranged in the following order. §2 gives a short excursion to nonlinear vibrations, §3 presents a kaleidoscope of fluid phenomena around a circular cylinder, §4 treats modern developments in vortex-induced vibrations very briefly, an alternative method of calculation is given in...
§5, then follow a discussion and conclusion in the last Section. In order to keep this paper to a manageable size, the reader is referred to the original papers for all mathematical derivations.

Before embarking on our guided tour, we wish to list some basic facts pertaining to aeolian vibration of overhead lines.

Conductor diameter \( D = 2.5 - 5 \text{ cm} \) (1 - 2 m ).

Wind velocity \( U = 1 - 7 \text{ m/s} \) (2 - 15 mile/h).

Vibration amplitude \( A_v = 0.01 - 1 \text{ m} \).

Using the above data and \( \nu = 0.1322 \text{ cm}^2/\text{s} \) for the kinematic viscosity of the standard air, and assuming a Strouhal number equal to 0.2, it is easy to establish our range of interest as follows:

Frequency \( f = 4 - 56 \text{ Hz} \).

Reynolds number \( Re = 1.9 \times 10^3 - 2.7 \times 10^4 \).

2 A SHORT EXCURSION TO NONLINEAR VIBRATIONS

Detailed treatment of nonlinear mechanics can be found in the excellent books by Minorsky [11] and Stoker [12]. We shall bring out four salient points of interest in the realm of nonlinear mechanics pertinent to aeolian vibrations.

2.1 Synchronization (lock-in, frequency entrainment)

The synchronization effect can readily be observed in electronic circuits. If one applies an external voltage of frequency \( \omega \) to the grid of a triode oscillating, say, with frequency \( \omega_0 \), one observes the 'beats' of the two frequencies. If the frequency \( \omega \) approaches the frequency \( \omega_0 \), the frequency of the beats decreases, but this happens only up to a certain value of the difference \( |\omega - \omega_0| \) after which the beats disappear suddenly and there remains only the frequency \( \omega \). Everything happens as if the self-excited frequency \( \omega_0 \) were 'entrained' by the external frequency \( \omega \). Figure 1 shows the difference \( |\omega - \omega_0| \) plotted against \( \omega \). On the basis of the linear theory the variation of \( |\omega - \omega_0| \) should follow the path ABCDE, whereas in reality it follows the path ABB'CD'DE.

The phenomenon of synchronization is, perhaps, the best known among other nonlinear phenomena. It has been observed not only in electrical systems but also in other systems. For instance, in the case of the vortex-induced vibration of the oscillating cylinder, \( \omega_0 \) would be the Strouhal frequency and \( \omega \) would be the oscillating frequency of the cylinder.

2.2 Frequency demultiplication, subharmonics

The existence of harmonics is well known for the linear system. For a nonlinear system subjected to an 'impure' disturbance, the response occurs not only with harmonics but also with an additional spectrum of the so-called combination tones. Some of the combination tones possess frequencies which are equal to a fraction of the fundamental frequency of the disturbance, \( \omega/n \), where \( n \) is an integer. These lower frequencies are called subharmonics, and the process by which they are obtained is called frequency demultiplication or division.

2.3 Jump phenomenon

Jump phenomena are demonstrated in Fig 2. If one starts with \( \omega = \omega_1 \) and increases the frequency, the amplitude \( a \) follows the stable branch AB. At \( \omega = \omega_2 \) (point B), the am-

Fig 1 \( |\omega - \omega_0| \) plotted against \( \omega \) (after Minorsky [11]).

Fig 2 Jump phenomena (after Minorsky [11]).
plitude drops suddenly on to the lower branch CD (point M) and follows it (from M to D) if \( \omega \) continues to increase. If, however, \( \omega \) decreases (say, from \( \omega = \omega_2 \)), the lower branch will be followed up to C, at this point there will be an upward jump CN on to the stable branch AB. The jump phenomenon is associated with the hysteresis of the system behavior and is also related to the theory of bifurcations.

2.4 Limit cycles of Poincaré

Consider the following van der Pol equation

\[
\frac{d^2x}{dt^2} - \mu(1-x^2) \frac{dx}{dt} + x = 0
\]

where \( 0 < \mu < 1 \). It is easily seen that eqn. (1) provides negative damping for \( x < 1 \) and positive damping for \( x > 1 \) if \( \frac{dx}{dt} \) remains positive. Let us introduce a new dependent variable \( y \) such that the second-order van der Pol equation is transformed into a set of two first-order differential equations

\[
\frac{dx}{dt} = y \\
\frac{dy}{dt} = \mu(1-x^2)y - x
\]

Equations (2a) and (2b) are the parametric equations of the phase trajectories of the van der Pol equation (1). They are said to be autonomous because they do not depend explicitly on the time. If (2b) is divided by (2a), the equation of the phase trajectory is obtained. This equation has the form

\[
\frac{dy}{dx} = \mu(1-x^2) - \frac{x}{y}
\]

The \( xy \) plane, on which the phase trajectory of eqn (3) appears, is called the phase plane.

Poincaré showed that the differential equations of the form

\[
\dot{x} = X(x,y); \quad \dot{y} = Y(x,y)
\]

occasionally admit special solutions represented by closed curves in the phase plane, which he calls limit cycles. A limit cycle is a closed trajectory (hence the trajectory of a periodic solution) such that no trajectory sufficiently near it is also closed. In other words, a limit cycle is an isolated trajectory. Every trajectory beginning sufficiently near a limit cycle approaches it either for \( t \to \infty \) or for \( t \to -\infty \), that is, it either winds itself upon the limit cycle, or unwinds from it. If all nearby trajectories approach a limit cycle \( C \) as \( t \to \infty \), we say that \( C \) is stable (Fig. 3(a)), if they approach \( C \) as \( t \to -\infty \), we say that \( C \) is unstable (Fig. 3(b)). If the trajectories on one side of \( C \) approach it while those on the other side depart from it, we sometimes say that \( C \) is semi-stable (Fig. 3(c)), although \( C \) must be considered unstable in practice.

Limit cycles, and in particular stable limit cycles, are fundamental in the theory of oscillations of nonlinear, nonconservative systems — the only kinds of systems in which they can arise. A stable limit cycle represents a stable stationary oscillation of a physical system. An example is shown in Fig. 4 in which both the outside and inside phase trajectories of eqn (3) for \( \mu = 0.1 \) wind upon the limit cycle, which happens to resemble a circle in this case. Figure 4 is taken from a paper on relaxation oscillations by van der Pol [13].
The problem of the periodic wake behind a cylinder in a moving fluid is of both historical and current interest. The fluid dynamics of flow around a circular cylinder is, indeed, very difficult and complex. Although much has been written on this subject, yet it is not fully understood. We refer to two standard treatises [14, 15] and two well-known review papers [16, 17] for details.

For small Mach number flow, the periodic wake of a smooth, circular cylinder depends only upon the Reynolds number. Figure 5 depicts the major regimes of vortex shedding from a circular cylinder as the Reynolds number increases monotonically. At very low Reynolds numbers based on cylinder diameter, the flow does not separate (Fig 5(a)). As Re is increased, the boundary layer separates symmetrically from the two sides at the positions S, S, and a fixed pair of Foppl vortices rotating in opposite directions is formed immediately beyond the cylinder (Fig 5(b)). As Re is further increased, the vortices elongate until one of the vortices breaks away and a periodic wake is formed. Up to a Reynolds number of approximately 150, the staggered vortex street is laminar and the two points of separation S, S, though not shown, are shifted somewhat forward (Fig 5(c)). At Re = 300, the vortex street is turbulent, and it degenerates into fully turbulent flow beyond approximately 50 diameters downstream of the cylinder. So far, the boundary layer has been laminar, but it changes into turbulent in the neighborhood of Re = 3 × 10^5, depending upon free stream turbulence and surface roughness. The Reynolds number range of 300 to approximately 3 × 10^5 has been called the subcritical range, within which the shedding occurs at a well-defined frequency. Just prior to the transition, the two points of separation S, S move to the most forward position (Fig 5(d)). At the transition Reynolds number, the cylinder drag drops sharply (see Fig 6). The vortex shedding is disorganized with a broad band of shedding frequencies, the two points of separation S, S move backwards, and the wake is much narrower (Fig 5(e)). At a higher, supercritical Reynolds number range, it has been found recently [19, 20] that the vortex street re-establishes itself.

Figure 6 shows the variation of C_D (drag coefficient) and of Strouhal number (St = fD/U) with Reynolds number (Re = UD/ν) of a circular cylinder with its axis perpendicular to the flow. It is seen that the drag coefficient drops suddenly when Re is somewhere between 10^5 and 5 × 10^5, the exact value of this 'transition' Reynolds number zone, or the region of 'critical' Reynolds number, Re_0, depends on the surface roughness, the free stream turbulence, etc. Prior to this transition, i.e., Re < Re_0, as shown previously in Fig 5(d), the boundary layer is laminar, the
two points of flow separation are in the most forward position, and the wake near the cylinder has a clear periodic structure with a dominant Strouhal frequency \( \text{At} \) and after this transition, \( \text{Re} > \text{Re}_\tau \), as demonstrated in Fig 5(e), the boundary layer becomes turbulent, the two separation points of the boundary layer move backward, and the wake shrinks and diffuses with a broad band spectrum Henceforth we shall speak of the Reynolds number regions below or above the transition zone as 'subcritical (laminar)' or 'supercritical (turbulent)', respectively.

**Fig 7 Evolution of vortex shedding frequency with flow velocity over oscillating cylinder (after Simu and Scanlan [22])**

4 MODERN DEVELOPMENT IN VORTEX-INDUCED VIBRATIONS

Vortex-induced vibrations are usually ascribed to vortex shedding, as mentioned in the Introduction For transmission lines, they are known as aeolian vibrations In recent years, there has been intensive research work done on the subject of vortex-induced vibrations because of its wide engineering applications, which range from antennas, bridges, heat exchanger tubes, masts, marine cables, offshore structures, space vehicles, stacks, and towers to transmission lines However, it is not the objective of this Section to list all pertinent literature on this subject, nor can it reproduce mathematical derivations and experimental details In short, we try to digest modern developments in vortex-induced vibrations and to present them in a very condensed form For those who desire more details, we are fortunate to have two excellent books which have been published recently [21, 22]

An up-to-date account of flow around a stationary cylinder has been given in the preceding Section Naturally, the next question to ask is what will happen to flow around an oscillating cylinder The strange fact, found by many experimentalists, is that vibrating cylinders produce streets of straight vortices with great regularity, whereas the same cylinders at rest produce slantwise vortex shedding. The effect of cylinder oscillation near the Strouhal frequency, normal to the flow and of interest to us, can be briefly explained as follows

When the cylinder oscillating frequency approaches the Strouhal frequency, a lock-in phenomenon suddenly appears. During lock-in, or synchronization, as shown in Fig 7, the Strouhal frequency is 'entrained' by the cylinder oscillating frequency which controls the vortex-shedding formation even when variations in flow velocity shift the nominal Strouhal frequency away from the cylinder oscillating frequency by a few per cent [23 - 29] One pertinent question concerning the frequency entrainment is whether there is any difference between self-excited and forced oscillating cylinders The succinct answer is no, based on a recent investigation [30] where two circular cylinders of equal diameter were used for experimentation one was tuned to self-excite under the influence of lift forces and the other was a rigid cylinder mounted on a shaker and forced to vibrate sinusoidally normal to the mean flow direction under conditions that duplicated the Reynolds number, Strouhal number, motion amplitude, and relative frequency of the tuned cylinder This lock-in effect can also be produced if the cylinder oscillating frequency is equal to a submultiple of the Strouhal frequency, i.e. subharmonics [23, 29] When the cylinder frequency varies around the lock-in zone, jump (hysteresis) phenomena appear [23, 24, 28] It is also observed that during lock-in, the amplitude of oscillation attains some fraction, rarely exceeding a half, of the cylinder diameter This self-limiting amplitude phenomenon can be explained on the following physical grounds As the amplitude of cylinder vibration is increased beyond \( \frac{D}{2} \), the symmetric pattern of alternately spaced vortices begins to break up It can be seen in Fig 8 that at an amplitude of \( 1D \) and \( \text{Re} = 190 \), three vortices are formed per cycle of vibration instead of the stable pattern of two
Fig 8 The vortex street behind a cylinder vibrating transverse to the mean flow at resonance with vortex shedding Re=190 (a) A stable, staggered vortex street (b) An unstable pattern with three vortices per cycle of vibration (after Griffin and Ramberg [31])

The break-up of a symmetric vortex shedding implies that the vortex-induced forces acting on a cylinder are self-limiting at a cylinder vibration amplitude around one diameter. To sum up, all experimental evidence reveals that the system consisting of a cylinder and a periodic wake behaves like a nonlinear self-excited oscillator under forced vibration, whereby the wake is the oscillator and the cylinder is forced to vibrate.

Theoretically, vortex-induced vibration is a branch of aero(hydro)elasticity, which is concerned with the study of phenomena wherein aerodynamic (hydrodynamic) forces and mechanical motions interact significantly. Based on a theoretical viewpoint, it would be very desirable to predict analytically the amplitude of vortex-induced vibration using surface pressures on the cylinder obtained from an analysis of the flow field. Ideally, one would solve the time-dependent Navier–Stokes equations in the presence of the vibrating cylinder, the flow separation and vortex formation would emerge naturally from the solution, and the pressure and shear loading on the cylinder surface would provide the forcing function for the coupled cylinder motion. Unfortunately, such a grandiose scheme of integrated analysis of both the flow field and the cylinder motion is not available. Naturally, the next best is to construct empirical models, based on experimental data, to describe the cylinder–fluid interaction. While these models do not solve the time-dependent Navier–Stokes equation, they do incorporate many of the dynamic effects that have been observed experimentally. The nature of self-excited vortex shedding suggests that the fluid behavior might be modeled by a simple, nonlinear oscillator. This idea was first conceived by Bishop and Hassan [23]. However, a lift oscillator model, which has achieved a certain degree of realism and acceptance, was first presented by Hartlen and Curne [32]. This was later modified by others [25, 33, 34]. The main elements of this model, as summarized in ref 35, are presented here. Let $D$ be the cylinder diameter. Then, with the notations $\omega_1^2 = k/m$ and $c/m = \zeta$, $\zeta$ being the ratio of damping $c$ to its critical value, the equation of motion of the cylinder may be written as

$$\frac{y}{D} + 2\zeta \omega_1 \frac{\dot{y}}{D} + \omega_1^2 \frac{y}{D} = \frac{\rho U^2}{2m} C_L$$

where $C_L$ is a time-varying lift coefficient. In particular, $C_L$ is assumed to satisfy an oscillation equation with the characteristics of a van der Pol oscillator [11] that is linked to the velocity of motion as follows

$$C_L + a_1 \dot{C}_L + a_2 \ddot{C}_L + a_3 C_L = a_4 \dot{y}$$

where $a_1, a_2, a_3, a_4$ are constants to be evaluated from experiment. This assumption is suggested by similarities between the behavior of the system being studied and that of the van der Pol oscillator, which exhibits low damping at low amplitudes and strong damping at high amplitudes.

Under the particular conditions of the problem, it is found expedient [35] to modify the oscillator equation to the following special form...
\[ C_L + \omega_s^2 C_L = \left[ C_{L0}^2 - C_L^2 - \frac{\dot{C}_L}{\omega_s} \right]^2 \times \]
\[ \times (\omega_s G \dot{C}_L - \omega_s^2 H C_L) = \omega_s F \frac{\dot{y}}{D} \]  

(7)

where \( \omega_s = 2\pi f_s \) is the Strouhal circular oscillation frequency defined by \( f_s D/U = \text{St} \), and \( C_{L0}, G, H, \) and \( F \) are parameters to be fitted according to the experimental results. Note that when \( \dot{y} = 0 \) (stationary cylinder), the solution for \( C_L \) is approximated by \( C_L = C_{L0} \sin(\omega_s t) \). Equations (5) and (7) constitute a coupled nonlinear pair. Since this model is valid only for an elastically supported cylinder (Hartlen-Curne), we shall not pursue it further. Instead, we prefer another model (Iwan-Blevins) which yields results for immediate application to aeolian vibrations.

The wake oscillator model advanced by Iwan and Blevins [36] is developed from the integral momentum equation for a control volume, as depicted in Fig 9, inside which a cylinder is located and in which a vortex street is formed. There are many basic similarities between the Iwan-Blevins model and the Hartlen-Curne model.

According to Blevins [21] the wake oscillator model predicts that an elastically mounted cylinder will exhibit large amplitudes of oscillation as the vortex-shedding frequency approaches the natural frequency of the cylinder. The response of the cylinder is a function of the ratio of the shedding frequency to the natural frequency of the cylinder structure, the structural viscous damping, and the ratio of the displaced fluid mass to the mass of the cylinder.

The model exhibits an entrainment effect, i.e., the frequency of vortex shedding from the elastically mounted cylinder is entrained by the natural frequency of structural oscillation. The model predicts that the entrainment effect will increase with decreasing structural damping, this is confirmed by experimental results. The model also predicts that the entrainment effect will increase with the ratio of the displaced fluid mass to the cylinder mass, except in the case of very small values of structural damping.

The peak resonant cylinder amplitude of the model can be expressed in terms of a single variable called reduced damping \( \text{St} = 2m(2\pi f_s)/\rho D^2 \). The peak resonant cylinder amplitude increases with decreasing reduced damping until a maximum limiting amplitude is reached. Iwan [10] showed that the model can be used to predict the resonant amplitude of elastic structures composed of circular cylindrical cross-sections. His results, shown in Fig 10, are limited to the subcritical Re range \( 2 \times 10^2 \rightarrow 2 \times 10^5 \), where the input experimental data were obtained. The equation of this curve is

\[ A_y = \frac{0.07 \gamma}{(\beta + 1.9)(\text{St})^2} \left[ 0.30 + \frac{0.72}{(\beta + 1.9)(\text{St})} \right]^{1/2} \]

(8)

where \( A_y \) is the maximum amplitude along the span for a particular mode corresponding to \( \gamma \). The values of \( \gamma \) for different structural elements are listed in Fig 10.
Two regions of Fig 10 are of particular interest. First, as structural damping approaches zero, the model predicts that the vortex-induced vibration reaches a maximum limiting amplitude between one and two diameters. This limit is produced by the reduction in the exciting component of fluid force with increasing amplitude, possibly caused by the breakdown of vortex shedding observed by Griffin [31] and shown in Fig 8. Second, the model predictions are well above the experimental data for amplitudes of the order of 0.1 diameter or less. This over-prediction is due to the use of two-dimensional analysis, in which the spanwise correlation is tacitly assumed to be unity. In fact the three-dimensional effect cannot be ignored, especially for low amplitudes of vibration around 0.1 diameter or less. Koopman [27] and Toebes [29] found that the cylinder vibration could enhance the spanwise correction. Figure 11, though obtained for an elastically supported cylinder, could serve as a sort of correction chart for low amplitudes of vibration of the order of 1/8 diameter or less.

5 AN ALTERNATIVE METHOD OF CALCULATION

In §4, we present two models, namely, the Iwan-Blevins model and the Hartlen-Currie model, which share many similarities. Since the Hartlen-Currie model is valid only for an elastically supported cylinder, we prefer the Iwan-Blevins model which has much wider applications. Equation (8), which is derived by Iwan [10] based on the Iwan-Blevins model, is valid not only for an elastically supported cylinder but also for elastic structures among which is the string. Equation (8) is plotted in Fig 10, along with experimental data for a rigid cylinder, a pivoted rod, and a cable. The vertical axis of Fig 10 represents the normalized maximum amplitude of response, while the horizontal axis represents the reduced damping, \( \gamma \), which is the mass ratio damping parameter. By using different form factors \( \gamma \), which are tabulated in Fig 10, one can obtain the maximum amplitude for different structural elements.

For aeolian vibrations, let us set the form factor \( \gamma = 1.155 \) and the Strouhal number \( St = 0.2 \). Then eqn (8) simplifies to

\[
\frac{A_y}{D} = \frac{2.02}{(\delta_x + 1.9)} \left[ \frac{0.3 + 3.6}{\delta_x + 1.9} \right]^{1/2}
\]

where \( A_y \) is the maximum amplitude of vibration in cm, \( D \) is the conductor diameter in cm, \( \delta_x \) is the reduced damping = \( 2m(2\pi\xi)/\rho D^2 \), \( \xi = c/c_a \) is the fraction of critical damping (dimensionless), \( \rho \) is the standard air density = \( 1.293 \times 10^{-3} \) g/m³, and \( m \) is the mass per unit length of conductor in kg/m.

Equation (9), which is one of the fruitful results derived from recent advances in engineering science, provides an alternative and simple method of calculating the aeolian vibration amplitude of a single conductor, provided that its fraction of critical damping \( \xi \) is known.

For example, let us take a Bersimis conductor which has \( D = 3.5 \) cm and \( m = 2.177 \) kg/m, and assume two typical values of \( \xi \), i.e., \( 10^{-2} \) and \( 10^{-3} \). Substituting these data into eqns (9) and (10), we obtain

\[
\begin{align*}
\xi &= 10^{-2}, \quad \delta_x = 1.727 \times 10^2, \quad \frac{A_y}{D} = 6.46 \times 10^{-4} \\
\xi &= 10^{-3}, \quad \delta_x = 1.727 \times 10^1, \quad \frac{A_y}{D} = 7.36 \times 10^{-2}
\end{align*}
\]

So far, we have demonstrated that the alternative method is able to compute the amplitude of aeolian vibration of single conductors provided that the dimensionless damping \( \xi \) is known. Therefore the question of how to obtain the dimensionless damping \( \xi \) is certainly pertinent. However, the answer...
to this question is not easy owing to the fact that it is a subject of much research nowadays and there are divergent opinions on it among researchers. We shall elaborate on it in the next Section.

6 DISCUSSION AND CONCLUSION

Equation (9) is limited to the subcritical Re range $2 \times 10^2 - 2 \times 10^5$, where the input experimental data are obtained. Fortunately, as shown in §1, the Re range of interest for aeolian vibrations is from $1.9 \times 10^3$ to $2.7 \times 10^4$. Furthermore, the transition Re for smooth cylinders is in the region of $1 \times 10^5 - 5 \times 10^5$, as shown in Fig 6, whereas the transition Re for stranded conductors is around $2.82 \times 10^4$ [42]. Therefore, from a fluid mechanics viewpoint, the use of eqn (9) for aeolian vibrations appears to be justified.

The conventional way of calculating the amplitude of aeolian vibration, as typified by Claren [8] and Hard in ref 9, is through power balance. However, there is an uncertainty factor of 2 pertaining to the wind power, as shown in Fig 3-3 of Hard's paper. This uncertainty is circumvented by the present method, for eqn (9) does not require information on the wind power. Nevertheless, in common with conventional methods, the present method does need information on damping; i.e., $\ddot{x}$.

Hard used a graphical method. A set of dissipation power curves with frequencies as parameters, which were obtained experimentally in the laboratory, intersect the wind power curve, thus yielding a set of vibration amplitudes corresponding to each frequency. Claren employed an analytical method with lengthy calculations. In brief, he stated that the dissipation power is a function of conductor diameter and tension and is proportional to $\lambda^{-3}$, where $\lambda$ is the wavelength.

By and large, there are two ways to obtain the damping information. One is through experimental means such as used by Hard in ref 9. The other is through analytical means such as employed by Claren [8]. In both cases cited, the dependence of damping on the vibration amplitude was not taken into consideration. Therefore, there appears to be room for improvement. As pointed out in §5, the question of obtaining damping information is, indeed, a loaded one. It is certainly beyond the scope of this paper to settle this question. The best we can do is to refer the reader to the original publications for details. It should be noted that both the conventional and the alternative method require correct damping information.

In a very recent paper, Noiseux [43] proposed a formulation of the internal losses of a conductor based on a complex flexural rigidity. His model takes care of the amplitude dependence and separates the free-field and the near-field losses according to their relative importance. According to Noiseux, his theoretical findings are in agreement with experimental observations by others, but are in variance with Claren's statement that the dissipation power is proportional to $\lambda^{-3}$. Therefore, we shall rely on his paper for damping information. Owing to space limitation, no attempts are made here to reproduce his mathematical derivations. Instead, we use his end results, from which Fig 12 is taken. Since the paper [43] may not be readily available outside Canada, a few words of explanation are in order. The separation of the free-field (FF) and the near-field (NF) losses and the dependence of damping on the amplitude of vibration for a Bersimis conductor are clearly demonstrated in Fig 12, in which the vertical axis represents the loss factor $\eta_c$ and the
horizontal axis represents the frequency with the amplitudes of vibration as parameters. It is to be noted that the loss factor $\eta_c$ is twice the fraction of critical damping, i.e., $\eta_c = 2\eta$. There are three solid curves corresponding to three vibration amplitudes, namely, 15', 7.5', and 3.75' respectively. At low frequencies (around 1 Hz), the near-field losses dominate, as shown by three dashed horizontal asymptotes. At high frequencies (around 50 Hz), the far-field losses are dominating, as demonstrated by the other three dashed asymptotes. Claren's method also appears in Fig. 12 as curve 'C', which, although ignoring the effect of the vibration amplitude, does provide approximate damping information from 4 Hz and above.

The use of 'slope' to represent the amplitude of vibration in Fig. 12 does require special explanation. For a clamped-clamped Bersimis conductor with $m = 2177$ kg/m, $L = 61$ m, and $T_0 = 3.36 \times 10^4$ N, as given in Fig. 12, it can be shown (see, for instance, ref. 6, p. 139) that for the $n$th mode there are $n$ loops (half sine curves) between two ends with natural frequency $f_n$, wavelength $\lambda_n$, and vibration amplitude $(A_y)_n$ as follows:

$$f_n = \frac{n}{2L} \left( \frac{T_0}{m} \right)^{1/2} = \frac{n}{2 \times 61} \left( \frac{3.36 \times 10^4}{2177} \right)^{1/2}$$

$$= n \text{ Hz}$$  \hspace{1cm} (11a)

$$\lambda_n = \frac{2L}{n} = \frac{2 \times 61}{n} = \frac{122}{n} \text{ m}$$  \hspace{1cm} (11b)

$$(A_y)_n = \frac{\lambda_n}{4} \sin \theta \approx \frac{\lambda_n}{4} \frac{\theta}{n} = \frac{122}{4n} \theta \text{ cm}$$  \hspace{1cm} (11c)

where $\theta$ is the angle of the slope of the above-mentioned half sine curves at its nodes. The 'slope' used in Fig. 12, which is taken from ref. 43, actually means the angle $\theta$ in units of minutes. Let us perform a sample calculation for $n = 4$. Then eqns (11) yield:

$$f = 4 \text{ Hz}$$  \hspace{1cm} (12a)

$$\lambda = 30.5 \text{ m}$$  \hspace{1cm} (12b)

$$A_y = 762.5 \theta \text{ cm}$$  \hspace{1cm} (12c)

where $\theta$ is in radians. The 'slopes', given in Fig. 12, of 15', 7.5', and 3.75' correspond respectively to $\theta = 4.36 \times 10^{-3}$, $2.18 \times 10^{-3}$, and $1.09 \times 10^{-3}$ rad. Substituting the three values of $\theta$ just obtained into eqn (12c) yields $A_y = 3.32, 1.66, 0.83$ cm, which correspond to 'slope' = 15', 7.5', 3.75' for $f = 4$ Hz. Now, one can perform similar calculations to convert 'slope' into vibration amplitude for other frequencies, the end results of which are summarized in Table 1.

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<th>'Slope'</th>
<th>Frequency $f$ (Hz)</th>
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</thead>
<tbody>
<tr>
<td>15'</td>
<td>4 1 0 2 0 4 0 6 0</td>
</tr>
<tr>
<td>7.5'</td>
<td>1 6 6 0 6 6 5 0 3 3 3</td>
</tr>
<tr>
<td>3.75'</td>
<td>0 8 3 0 3 3 6 0 1 6 6 0 0 8 4 0 0 5 6</td>
</tr>
</tbody>
</table>

Then we proceed to calculate the vibration amplitude $A_y$ from eqn (9) with the aid of Fig. 12 at three different frequencies. After a few iterations*, the final results are listed in Table 2.

<table>
<thead>
<tr>
<th>Frequency $f$ (Hz)</th>
<th>Amplitude $A_y$ from eqn (9) (cm)</th>
<th>Amplitude $A_y$ from Table 1 (cm)</th>
<th>Corresponding slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.24</td>
<td>1.24</td>
<td>5.625'</td>
</tr>
<tr>
<td>10</td>
<td>0.58</td>
<td>0.61</td>
<td>6.938'</td>
</tr>
<tr>
<td>40</td>
<td>0.08</td>
<td>0.083</td>
<td>3.75'</td>
</tr>
</tbody>
</table>

In conclusion, the new method is not only of academic interest, but also useful for practical applications.

**NOMENCLATURE**

- $A_y$: vibration amplitude, zero to peak, cm
- $C$: damping constant
- $C_D$: drag coefficient
- $C_L$: lift coefficient
- $D$: drag

* A sample iteration (cut and try method) is given in the Appendix.
**Greek symbols**

- $\gamma$: form factor
- $\delta_r$: reduced damping, mass ratio damping parameter
- $\zeta$: fraction of critical damping, dimensionless
- $\eta_c$: loss factor
- $\theta$: angle of vibrating string slope at node
- $\lambda$: wavelength
- $\mu$: viscosity, damping constant
- $\nu$: kinematic viscosity
- $\rho$: density
- $\omega$: circular frequency

**Subscripts**

- $c$, $cr$: critical
- $n$: $n$th mode
- $s$: Strouhal
- $o$: resonant, ambient

### APPENDIX

**A sample iteration (cut and try)**

**1st try**

Refer to Fig 12. Read $\eta_c$ at $f = 4\text{ Hz}$ on the 3.75' curve. One obtains $\eta_c = 1.9 \times 10^{-4}$. Then $\zeta = \eta_c/2 = 0.95 \times 10^{-4}$

For a Bessons conductor, $m = 2.177\text{ kg/m}$, $D = 3.5\text{ cm}$, and standard air density $\rho = 1.293 \times 10^{-3}\text{ g/cm}^3$, then eqn (10) becomes

$$\delta_r = 2m(2\pi\zeta)/\rho D^2 = \frac{2(2.177)(10^3/10^2)(2\pi\zeta)}{1.293 \times 10^{-3}(3.5)^2}$$

$$= 1.727 \times 10^{4}\zeta \tag{A1}$$

Substituting $\zeta = 0.95 \times 10^{-4}$ into eqns (A1) and (9) yields

$$\delta_r = 1.727 \times 10^{4}(0.95 \times 10^{-4}) = 1.64$$

$$\frac{A_y}{D} = 2.02 \left[ \frac{0.30 + 3.6}{1.64 + 1.90} \right]^{1/2}$$

$$= 6.55 \times 10^{-1}$$

or $A_y = 0.655D = 2.29\text{ cm}$, which is much greater than $A_y = 0.83\text{ cm}$ as listed in Table 1 under row 3.75' and column 4 Hz. So we have to try again.

**2nd try**

Read $\eta_c$ at $f = 4\text{ Hz}$ on the 7.5' curve. One obtains $\eta_c = 6.5 \times 10^{-4}$. Then $\zeta = \eta_c/2 = 3.25 \times 10^{-4}$. Substituting $\zeta = 3.25 \times 10^{-4}$ into eqns (A1) and (9) yields

$$\delta_r = 1.727 \times 10^{4}(3.25 \times 10^{-4}) = 5.61$$

$$\frac{A_y}{D} = \frac{2.02}{(5.61 + 1.9)} \left[ \frac{0.30 + 3.6}{5.61 + 1.9} \right]^{1/2}$$

$$= 0.237$$

or $A_y = 0.237D = 0.83\text{ cm}$, which is much smaller than $A_y = 1.66$ found in Table 1 under row 7.5' and column 4 Hz.

**3rd try**

Since the first try ends in an overshoot and the second try results in an undershoot, let us try to average them.

From Table 1,

Average 'slope' = \[ \frac{3.75 + 7.5}{2} = 5.625' \]

Average $A_y = \frac{0.83 + 1.66}{2} = 1.24\text{ cm}$

Then, we have

Average $\zeta = \frac{0.95 + 3.25}{2} \times 10^{-4} = 2.1 \times 10^{-4}$

Substituting $\zeta = 2.1 \times 10^{-4}$ into eqns (A1) and (9) yields

$$\delta_r = 1.727 \times 10^{4}(2.1 \times 10^{-4}) = 3.63$$

$$\frac{A_y}{D} = \frac{2.02}{(3.63 + 1.90)} \left[ \frac{0.30 + 3.6}{3.63 + 1.90} \right]^{1/2}$$

$$= 0.356$$

$A_y = 0.356D = 1.24\text{ cm}$ which agrees with the tabulated value of 1.24 cm very well, thus ending our cut and try process. The value is tabulated along with its corresponding 'slope' of 5.625' in Table 2. Certainly, similar calculations can be performed for the other two rows.
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