A KALMAN FILTER APPROACH FOR GALLOPING
CONTROL OF A BRIDGE TOWER

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Abstract—An active optimal control scheme is presented for the control of the galloping related vibrations in the long span bridge tower. The Kalman filter technique has been used for the state variable estimation. The Kalman filter approach includes the effect of gust loading in the control algorithm by considering the statistical properties of the wind loading, resulting in a superior control algorithm than the one presented previously with an optimal observer. A dual Active Tuned Mass Damping (ATMD) system is suggested for the control of a bimodal galloping phenomenon due to the simultaneous galloping of the two closely spaced vibration modes. A comparative study is also carried out to observe the effectiveness of an Active Tuned Mass Damper over that of a Passive Tuned Mass Damper with the changing mass ratio of the TMD.

1. INTRODUCTION

Galloping is an aerodynamic instability, which is one of the special features of flexible and bluff structures such as the towers of long span bridges. Until recently galloping was defined as a single mode bending flutter of the fundamental mode of the structure [1, 2]. The results of recent research [3] showed that in some cases, where the fundamental mode is highly damped through such dampers as the Tuned Mass Damper, the higher modes start to galloping. It was also observed [4, 5] that in some special cases, where two structural frequencies are closely spaced, the galloping of one mode is accompanied by the galloping of other modes and hence more than one mode gallops simultaneously. This is known as a multi-mode galloping phenomenon. This kind of galloping behavior was found in the wind tunnel model testing of the Higashi Kobe Bridge tower. Such a galloping leads to an enormously large vibration, thus calling for its control.

Several efforts have been reported in the past to control these vibrations. Shiraishi et al. [6] proposed a shape modification to control these vibrations which was implemented in the actual bridge design. Alam et al. [7] suggested a robust active control scheme for the control of these vibrations. They argued that, considering the fact that an even more flexible and delicate structure was likely to result in the future, where an unprecedented architectural and aesthetical demand lead to a partially braced tower. The special architecture of this tower leads to two closely spaced natural frequencies and subsequently, multi-mode galloping. The control system comprised an optimal regulator augmented with an optimal observer to account for any uncertainty in the system parameter predictions and also to cater for the gust loading, as the gust loading, being random in nature, cannot be predicted beforehand. Their proposal was based solely on the observer measurement, while neglecting the gustiness in the wind. Furthermore, the type of controller was also omitted. In the present study the above two problems are discussed by first opting for a prediction-type Kalman filter approach for the estimation of state variables. The choice of Kalman filter makes the system more robust in the sense that it not only measures the observed response, but also includes gustiness in the wind into the system by considering the statistical properties of the wind velocities. This kind of prior information about the gust load makes this system superior to that of an active control system with an optimal observer. Secondly, for the controller an Active Tuned Mass Damping system (ATMD) is suggested. This system is particularly adopted to augment the beneficial dampening effect of the Passive Tuned Mass system (TMD) and supplement it with an active control. In order to show the advantage of an active control system such as an Active Tuned Mass Damper over that of a passive system such as Passive Tuned Mass Damper, the controller has been designed in two stages. First the two modes are tuned with two dampers. In the second stage an actuator is added for the desired control of the system. The performance of the two systems, i.e. the Passive TMD and the Active
TMD, is compared and as expected the performance of ATMD is found to be considerably better than that of passive TMD.

There is a limit to the TMD mass which can be added to the structure, especially in the futuristic structure, where it would be much taller than those at present and even a 1 or 2% increase in the structural mass would be enormous. In this study a comparison has also been made between the Passive and Active TMD’s performances with the changing TMD masses.

For the wind force modeling, it is assumed that the wind velocity is composed of two components, the average wind velocity and the fluctuating wind velocity. The average wind velocity component leads to the aerodynamic instability. This force is modeled by the Quasisteady theory \[I\] and assumed to be deterministic. The gust load due to the turbulence in the wind, which is purely a random phenomenon, is treated as the system noise and replaced by its statistical properties.

2. BASIC ANALYSIS

Consider an aerodynamically-bluff continuous body, like a bridge tower, subjected to a wind stream as shown in Fig. 1. The equation of motion perpendicular to the direction of the wind is given by

\[
m(z) \frac{\partial^2 y(z, t)}{\partial t^2} + c(z) \frac{\partial y(z, t)}{\partial t} + \frac{\partial^2}{\partial z^2} \left[ EI(z) \frac{\partial^2 y(z, t)}{\partial z^2} \right] = F_{aw}(z, t) + F_b(z, t) \tag{1}
\]

where \(m(z), c(z)\) and \(I(z)\) are the mass, damping and the moment of inertia of the structure per unit length, respectively. \(E\) is the modulus of elasticity of the structural material and \(y(z, t)\) is the displacement function. \(F_{aw}(z, t)\) and \(F_b(z, t)\) are the self-exciting and the buffeting forces per unit length, respectively. The displacement function \(y(z, t)\) is given as

\[
y(z, t) = \sum_{s=1}^{\infty} \Psi_s(z)q_s(t) \tag{2}
\]

where \(\Psi_s(z)\) and \(q_s(t)\) are the mode shape and the generalized co-ordinate for the \(s\)th mode. Substituting eqn (2) into eqn (1), multiplying the resulting equation by \(\Psi_s(z)\) and then integrating it over the full height \(l\), the inertial and the elastic forces on the left hand side of the equation are decoupled. Again considering the structural damping term to be linear, it is also decoupled. The buffeting force term on the right hand side of the equation is also assumed to be uncoupled [9], whereas the aerelastic force, which is discussed in the next sub-section, is a non linear term, which remains coupled. The equation of \(n\)th mode can be written as

\[
\int_0^l m(z)\Psi_n^2(z) \, dz \left[ \ddot{q}_n(t) + 2\beta_n \omega_n \dot{q}_n(t) + \omega_n^2 q_n(t) \right] = \int_0^l F_{aw}(z, t)\Psi_n(z) \, dz + \int_0^l F_b(z, t)\Psi_n(z) \, dz \tag{3}
\]

where \(\omega_n\) and \(\beta_n\) are the circular frequency and damping ratio of the \(n\)th mode, respectively. The selection of the total number of modes depends on their participation in the self-excited and gusty motion. For simplicity we have limited our analysis to only two modes. These modes are selected for the purpose of analysis. For instance, in this work to observe the multi-mode galloping phenomenon, we will take those two modes which are nearly equal or closely spaced. However the analysis can be extended to any number of modes. We have named the selected two modes as \(n = \zeta\) and \(\eta(\omega_\zeta \leq \omega_\eta)\). Hence the generalized equation of motion for the \(n\)th mode is given as

\[
\ddot{q}_n(t) + 2\beta_n \omega_n \dot{q}_n(t) + \omega_n^2 q_n(t) = F_{aw}(t) + F_b(t) \tag{4}
\]

here

\[
F_{aw}(t) = \frac{1}{\mu_n D_n} \int_0^l F_{aw}(z, t)\Psi_n(z) \, dz \tag{5a}
\]

\[
F_b(t) = \frac{1}{\mu_n D_n} \int_0^l F_b(z, t)\Psi_n(z) \, dz \tag{5b}
\]

where \(\mu_n = \int_0^l m(z)\Psi_n^2(z) \, dz / D_n\) and \(D_n = \int_0^l \Psi_n^2(z) \, dz\), \(n = \zeta, \eta\).

2.1. Self-exciting force

When some special structures with typical cross sectional shapes, such as the D-shape sections or the rectangular sections, are immersed in a fluid, a force is generated. This force depends on the relative motion of the body and the fluid. At a particular relative velocity, the generated forces excite the structural motion and are known as self-exciting forces. According to the quasi-steady aerodynamic theory, the self-exciting forces can be modeled as [1]

\[
F_{aw}(z, t) = \frac{1}{2} \rho h V(t)^2 \sum_{j=1}^{\infty} A_j \left( \frac{\partial^2 y_j(z, t)}{\partial t^2} \right) V(z) \tag{6}
\]

\[
j = 1, 3, 5, 7
\]
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where \( \rho \) is the air density, and \( h \) is the characteristic dimension of the structure. Conventionally it is taken as the dimension of the lifting body perpendicular to the wind direction. Following the convention, \( h \) is defined as the dimension of the leg from the tower perpendicular to the wind, as shown in Fig. 1. \( V(z) \) is the mean wind velocity at height \( z \). \( A_1 \) is the aerodynamic coefficient. The mean velocity of air \( V(z) \) is given as

\[
V(z) = V_0 v(z) = V_0 \left( \frac{z}{z_0} \right) ^\rho
\]  

(7)

where \( V_0 \) is the mean wind velocity at the reference height and \( z_0 \) is the height of the reference point usually taken as \( z_0 = 10 \) m. \( \rho \) is a constant depending on the topographic condition. Substituting eqns (2), (6) and (7) into eqn (5a), and considering only two modes, i.e. \( n = \zeta \) and \( \eta \), the generalized self-exciting force can be found as

\[
F_m(t) = \frac{\rho h}{2 \mu_n D_n} \left[ A_1 V_0 \int_0^1 v(z) \Psi_m(z) \right.
\]

\[
\times \left( \sum_{n=\zeta,\eta} \frac{1}{n} \Psi_m(z) \Psi_m(t) \right) dz + A_2 V_0^{-1}
\]

\[
\times \left[ \sum_{n=\zeta,\eta} \psi_m(z) q_{m,t}(t) \right] \right] dz + A_7 V_0^{-5}
\]

\[
\left[ \sum_{n=\zeta,\eta} \frac{1}{n} \psi_m(z) \psi_m(t) \right] \right] \right] \right] \right]
\]

(8)

Equation 8 can be written as

\[
F_m(t) = \frac{2 \kappa_n}{h} f_{m}(\tilde{q})
\]

(9)

where

\[
f_{m}(\tilde{q}) = A_1 V_0 C_{a,n} \tilde{q}_{m,1}(t) + A_3 V_0^{-1} \sum_{k=2}^{5} \left( \frac{3}{k-2} \right)
\]

\[
\times (C_{a,n} \tilde{q}_{m,2}^{k-1}(t) \tilde{q}_{m,2}^{-2}(t) + A_3 V_0^{-1} \sum_{k=6}^{11}
\]

\[
\times \left( \frac{5}{k-6} \right) C_{a,n} \tilde{q}_{m,3}^{k-6}(t) \tilde{q}_{m,3}^{-4}(t) + A_8 V_0^{-5}
\]

\[
\times \left( \frac{7}{k-12} \right) C_{a,n} \tilde{q}_{m,4}^{k-12}(t) \tilde{q}_{m,4}^{-8}(t)
\]

and \( \kappa_n = \rho h^2/4 \mu_n, n = \zeta \) and \( \eta \), where \( n \) and \( k \) describe the mode number and the coefficient number, respectively, whereas the coefficients \( C_{a,n} \) are given as follows

\[
C_{a,n} = \int_0^1 v(z) \Psi_m^\zeta(z) \Psi_m^\eta(z) dz / D_n, \quad k = 1
\]

\[
C_{a,n} = \int_0^1 v(z) \Psi_m^{\zeta-1} \Psi_m^{\eta-1} dz / D_n, \quad k = 2, 3, 4, 5
\]

\[
C_{a,n} = \int_0^1 v(z) \Psi_m^{\zeta-2} \Psi_m^{\eta-2} dz / D_n, \quad k = 6, 7, \ldots, 11
\]

\[
C_{a,n} = \int_0^1 v(z) \Psi_m^{\zeta-3} \Psi_m^{\eta-3} dz / D_n, \quad k = 12, 13, \ldots, 19
\]

where \( n = \zeta \to \zeta = 1 \) and \( n = \eta \to \xi = 2 \).

2.2. Buffeting force

The fluctuation in the wind velocities results in a force which is random in nature and is known as the buffeting force. From eqn (5b) the mean square value of buffeting force for the \( n \)th mode is given as [9]

\[
F_b^2(t) = \frac{1}{\mu_n^2 D_n} \int_0^1 \Psi_m(z, t) \Psi_n(z', t) dz dz'
\]

(10)

where a bar over a quantity denotes the time averages. \( F_b(z, t) \) represents the covariance of the buffeting force at points \( z \) and \( z' \). The buffeting force \( F_b(z, t) \) can be modeled as [10]

\[
F_b(z, t) = \frac{1}{\mu_n^2 D_n} \int_0^1 \Psi_m(z, t) \Psi_n(z', t) dz dz' \]

(11)

where \( \bar{v}(z, t) \) and \( \bar{w}(z, t) \) are along and across wind fluctuating velocities. \( C_l \) is the lift coefficient and \( C_{l}' = dC_l/dz \), \( z \) is the angle of attack. Contrary to the conventional way [10], where the characteristic dimension is taken as the dimension of the lifting surface \( b \), here for the sake of consistency, it is again defined as \( h \). In other words we have assumed that the lift force is nondimensionalized by \( h \) instead of \( b \). Substituting eqns (7) and (11) into eqn (10) and neglecting the cross terms due to the along and across
wind fluctuating velocities \[10\], we get

\[
\overline{F_{\text{w}}(t)} = \frac{1}{\mu_i D_i^2} \int_{0}^{\infty} \int_{0}^{\infty} \left( \frac{1}{2} p h V_0 \right) v(z) v(z') \left( (2 C_L)^2 \overline{v(z, t) v(z', t)} + (C_L)^2 \overline{v(z, t) v(z', t)} \right) \times \Psi_n(z) \Psi_n(z') \mathrm{d} z \mathrm{d} z'.
\]

The covariance of the along-wind fluctuating velocity at points \(z\) and \(z'\) in eqn (12) is given as

\[
\overline{w(z, t) w(z', t)} = 2 \int_{0}^{\infty} S_{w, w}(\omega) \times \exp \left( -k_i \frac{\omega}{2\pi V_0} \Delta z \right) \mathrm{d} \omega.
\]

Similarly

\[
\overline{w(z, t) w(z', t)} = 2 \int_{0}^{\infty} S_{w, w}(\omega) \times \exp \left( -k_i \frac{\omega}{2\pi V_0} \Delta z \right) \mathrm{d} \omega.
\]

where \(S_{w, w}(\omega)\) and \(S_{w, w}(\omega)\) are one sided spectra of the along and the across fluctuating wind components at the reference point, respectively and \(k_i\) is the decay factor. \(\Delta z\) is the separation of points \(z\) and \(z'\).

Substituting eqn (13) into eqn (12), we get

\[
\overline{F_{\text{w}}(t)} = \left( \frac{2k_i \Delta z}{h} \right)^2 \times \int_{0}^{\infty} \int_{0}^{\infty} v(z) v(z') \Psi_n(z) \Psi_n(z') \left( (2 V_0 C_L)^2 \right) \times S_{w, w}(\omega) \exp \left( -k_i \frac{\omega}{2\pi V_0} \Delta z \right) \mathrm{d} \omega \mathrm{d} z \mathrm{d} z'.
\]

Equation (14) can be written as

\[
\overline{F_{\text{w}}(t)} = \left( \frac{2k_i \Delta z}{h} \right)^2 \left( (2 V_0 C_L)^2 \right) \times \int_{0}^{\infty} \int_{0}^{\infty} S_{w, w}(\omega) \left| J_\nu(\omega) \right|^2 \mathrm{d} \omega
\]

\[
+ (V_0 C_L)^2 \int_{0}^{\infty} \int_{0}^{\infty} S_{w, w}(\omega) \left| J_\nu(\omega) \right|^2 \mathrm{d} \omega
\]
where $K_0$ is the surface drag coefficient. The power spectral density for the lateral velocity fluctuation is given as [13]

$$P_{w}(\omega, z) = \frac{15K_0 V_0^2 z}{2\pi V(z)[1 + 9.5\omega z/2\pi V(z)]^{3/2}}. \quad (21)$$

3. EQUATION OF MOTION WITH ACTIVE TUNED MASS DAMPERS

The vibration due to the aerodynamic and gust forces can be controlled by the addition of Tuned Mass Dampers (TMD) to the system. If these TMDs are just tuned with the natural frequencies of the structure, they act as passive TMDs. On the other hand, if their motion is regulated by the actuators, they become Active Tuned Mass Dampers (ATMD). In the text hereafter, Passive Tuned Mass Damper will be referred to simply as TMD and Active Tuned Mass Damper as ATMD. Since in this work we are confined to a two mode multi-mode galloping problem, two dampers are subsequently tuned with the two modes and are correspondingly provided with the two actuators. Let the position of these dampers be denoted by $\pi$ and $\sigma$. Equation (1) in the presence of dampers and the actuators can then be written as [14]

$$m(z) \ddot{Y}(z, t) + c(z) \dot{Y}(z, t) + s \omega \cdot W = F_{e}(t) + F_{m}(t). \quad (22)$$

Substituting eqn (23) into eqn (24) and rearranging the equation of motion for the $\pi$th damper can be written as

$$\ddot{Y}_\pi \left[ \left( \frac{1}{M_\pi} + \sum_{n=\pi}^{\sigma} \frac{\Psi_n(a_\pi)}{\mu_n D_n} \right)(K_\pi Y_\pi + C_\pi \dot{Y}_\pi) + \sum_{n=\pi}^{\sigma} \frac{\Psi_n(a_\pi)}{\mu_n D_n} 2\beta_n \omega_n q_\pi(t) - \sum_{n=\pi}^{\sigma} \sum_{j=\pi}^{\sigma} \left[ \frac{\Psi_n(a_\pi)}{\mu_n D_n} \right] \omega_j q_j(t) \right] = -\sum_{n=\pi}^{\sigma} (a_\pi)\left[ F_{e}(t) + F_{m}(t) \right] + \frac{1}{M_\pi} U_\pi \quad (25)$$

Similarly, the equation of motion for the $\sigma$th damper can be formulated.

4. STATE EQUATION FOR ACTIVE CONTROL

The two equations of motion corresponding to the two modes $\zeta$ and $\eta$ and the other two equations corresponding to the two dampers are arranged leading to state eqn (26) as given below

$$\dot{Q} + CQ + KQ = HU + S(F_\pi(\dot{q}_\pi) + F_\sigma(\dot{q}_\sigma)) \quad (26)$$

where $Q$ is a state vector of size $4 \times 1$. $C$ and $K$ are the damping and the stiffness matrices of size $4 \times 4$ and $S$ is a coefficient matrix of size $4 \times 2$. $H$ is a coefficient matrix also of size $4 \times 2$. $U$ is the control vector of size $2 \times 1$. $F_\pi(\dot{q}_\pi)$ and $F_\sigma(\dot{q}_\sigma)$ are the force
vectors of size $2 \times 1$. They are given as

$$Q = \begin{bmatrix} q_{i(t)} \\ q_{i(t)} \\ Y_i \\ Y_i \\ \end{bmatrix}, \quad F_i(\dot{q}_i) = \begin{bmatrix} F_{c}(\dot{q}_i) \\ F_{c}(\dot{q}_i) \\ \end{bmatrix}, \quad F_i(t) = \begin{bmatrix} F_{c}(t) \\ F_{c}(t) \\ \end{bmatrix}, \quad U = \begin{bmatrix} U_{i(t)} \\ U_{i(t)} \\ \end{bmatrix}$$

$$H = \begin{bmatrix} -\Psi_c(a_{dt})/\mu_D & -\Psi_c(a_{dt})/\mu_D \\ \left(\frac{1}{M_c} + \sum_{n=1}^{\infty} \Psi^2_n(a_{dt})/\mu_D \right) & \left(\frac{1}{M_c} + \sum_{n=1}^{\infty} \Psi^2_n(a_{dt})/\mu_D \right) \\ \sum_{n=1}^{\infty} \Psi_n(a_{dt})\Psi_n(a_{dt})/\mu_D \right) & \sum_{n=1}^{\infty} \Psi_n(a_{dt})\Psi_n(a_{dt})/\mu_D \right) \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\Psi_n(a_{dt}) & -\Psi_n(a_{dt}) \\ -\Psi_n(a_{dt}) & -\Psi_n(a_{dt}) \end{bmatrix}$$

Equation (26) is modified to a first order differential equation as follows,

$$Q^* = AQ^* + BU + D(F_i(\dot{q}_i) + F_i(t))$$

where $Q^*$ is a new state vector of size $8 \times 1$. $A$ is the coefficient matrix of size $8 \times 8$ and $B$ and $D$ are the coefficient matrices of size $8 \times 2$, which are given as

$$Q^* = \begin{bmatrix} Q \\ \dot{Q} \end{bmatrix}, \quad A = \begin{bmatrix} -C & -K \\ I & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} H \\ 0 \end{bmatrix} \text{ and } D = \begin{bmatrix} S \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 2\beta_1 \omega_c & 0 & (-\Psi_c(a_{dt})/\mu_D)C_x & -(\Psi_c(a_{dt})/\mu_D)C_x \\ 0 & 2\beta_1 \omega_q & (-\Psi_n(a_{dt})/\mu_D)C_x & -(\Psi_n(a_{dt})/\mu_D)C_x \\ -2\Psi_c(a_{dt})/\beta_1 \omega_c & -2\Psi_n(a_{dt})/\beta_1 \omega_q \left(\frac{1}{M_c} + \sum_{n=1}^{\infty} \Psi^2_n(a_{dt})/\mu_D \right)C_x & \left(\frac{1}{M_c} + \sum_{n=1}^{\infty} \Psi^2_n(a_{dt})/\mu_D \right)C_x \\ -2\Psi_c(a_{dt})/\beta_1 \omega_c & -2\Psi_n(a_{dt})/\beta_1 \omega_q \left(\sum_{n=1}^{\infty} \Psi_n(a_{dt})\Psi_n(a_{dt})/\mu_D \right)C_x & \left(\sum_{n=1}^{\infty} \Psi_n(a_{dt})\Psi_n(a_{dt})/\mu_D \right)C_x \end{bmatrix}$$

$$K = \begin{bmatrix} \omega_c^2 & 0 & (-\Psi_c(a_{dt})/\mu_D)K_x & -(\Psi_c(a_{dt})/\mu_D)K_x \\ 0 & \omega_q^2 & (-\Psi_n(a_{dt})/\mu_D)K_x & -(\Psi_n(a_{dt})/\mu_D)K_x \\ -\Psi_c(a_{dt})/\omega_c^2 & -\Psi_n(a_{dt})/\omega_q^2 \left(\frac{1}{M_c} + \sum_{n=1}^{\infty} \Psi^2_n(a_{dt})/\mu_D \right)K_x & \left(\frac{1}{M_c} + \sum_{n=1}^{\infty} \Psi^2_n(a_{dt})/\mu_D \right)K_x \\ -\Psi_c(a_{dt})/\omega_c^2 & -\Psi_n(a_{dt})/\omega_q^2 \left(\sum_{n=1}^{\infty} \Psi_n(a_{dt})\Psi_n(a_{dt})/\mu_D \right)K_x & \left(\sum_{n=1}^{\infty} \Psi_n(a_{dt})\Psi_n(a_{dt})/\mu_D \right)K_x \end{bmatrix}$$
Equation (27), which is a nonlinear equation because of the velocity dependent self exciting forces, can be linearized by assuming a time lag $\Delta t$ in the self-excited forcing function $i.e. F_i(q_i, \dot{q}_i) \approx F_i(q_i, \dot{q}_i - \Delta t)$ [7, 8] ($\Delta t$ is the sampling time used in the discretization of the differential equation to finite difference equation). With this consideration the state eqn (27) is modified as follows:

$$\dot{Q}^* = AQ^* + BU + D(F_i(q_i, \dot{q}_i) + F_o(t)).$$  \hspace{1cm} (28)$$

Discretization of eqn (28) results in the following linearized incremental state equation [15]

$$Q^*_{i+1} = A^*Q^*_i + C^*U_i + D^*F_{e,i-1} + D^*F_{b,i}.$$  \hspace{1cm} (29)$$

where

$$A^* = e^{A\Delta t}, \quad C^* = (e^{A\Delta t} - I)A^{-1}B$$

and

$$D^* = (e^{A\Delta t} - I)A^{-1}D.$$  \hspace{1cm} (30)$$

5. OPTIMAL CONTROL

Using the control force $U$ and the state variable $Q^*$ at each time step $i$, the objective function $J$ is specified as below:

$$J(Q^*, U) = \frac{1}{2} \sum_{i=0}^{\infty} (Q^*_i\dot{R}_Q Q^*_i + U_i\dot{R}_U U_i).$$  \hspace{1cm} (31)$$

The diagonal matrices $\dot{R}_Q$ and $\dot{R}_U$ are positive weight functions as represented by

$$\dot{R}_Q = r_Q I, \quad \dot{R}_U = r_U I.$$  \hspace{1cm} (32)$$

$J(Q^*, U)$ has its minimum value when the matrix $P$ satisfies the stationary Riccati eqn (32) as given below [15]. The nonlinear aerodynamic force term has been neglected in the minimization [7, 8].

$$P = \dot{R}_Q A^*PA^* A^*PC^* \times (\dot{R}_U + C^*PC^*)^{-1}C^*PA^*.$$  \hspace{1cm} (33)$$

The magnitude of optimal control force $U_i$ at each time step $i$ is given by [15]

$$U_i = -E_A Q^* - E_D F_{e,i-1}.$$  \hspace{1cm} (34)$$

In eqn (33), the buffeting force has been neglected as it is known a priori, while the coefficient matrices $E_A$ and $E_D$ are given by

$$E_A = (\dot{R}_U + C^*PC^*)^{-1}C^*PA^*$$

$$E_D = (\dot{R}_U + C^*PC^*)^{-1}C^*PD^*.$$  \hspace{1cm} (35)$$

6. KALMAN FILTER AS A STATE ESTIMATOR

In the state equation, eqn (29), the gust force is indeterministic and acts as a system noise. Let us consider that $k$ sensors are attached for measuring actual response $Y_i$, then the output equation is given as

$$Y_i = GQ^*_i + \epsilon_i.$$  \hspace{1cm} (36)$$

where $G$ is a $k \times 8$ matrix used for connecting the measured value to the state vector. $\epsilon_i$ is the measurement noise. Thus eqns (29) and (34) define the system state equation and the output equation. The term $D^*F_{b,i}$ represents the system noise. We assume that the initial state $Q^*_0$, the system noise $D^*F_{b,i}$ and the measurement noise $\epsilon_i$ satisfy the following conditions:

$$E[Q^*_0] = Q^*_0$$

$$E[D^*F_{b,i}] = 0$$

$$E[\epsilon_i] = 0$$

$$E[[Q^*_0 - Q^*_0] Q^*_0] = A_0$$

$$E[[D^*F_{b,i}] [D^*F_{b,i}]] = S^*_\delta \beta$$

$$E[[\epsilon_i] [\epsilon_i]] = R^*_\delta \beta$$

$$E[[Q^*_0 - Q^*_0] [D^*F_{b,i}]] = 0$$

$$E[[Q^*_0 - Q^*_0] [\epsilon_i]] = 0.$$  \hspace{1cm} (37)$$

We wish to determine the optimal state $Q^*_i$, such that

$$A_i e_i = E[\{e_{i+1}\} [e_{i+1}]]$$  \hspace{1cm} (38)$$

is minimum, where $e_i = Q^*_i - \dot{Q}^*_i$. Here $A_i$ is minimum implies that $x^T A_i x$ is minimum, where $x$ is an arbitrary vector.

6.1. Prediction-type Kalman filter

Let the prediction-type Kalman filter have the form [15]

$$\dot{Q}^*_{i+1} = A^*Q^*_i + C^*U_i + D^*F_{e,i-1} + L_i(Y_i - \dot{Y}_i).$$  \hspace{1cm} (39)$$

where $L_i$ is an observer gain matrix. We now seek a matrix $L_i$ such that $E[[\{e_{i+1}\} [e_{i+1}]]$ is minimum. It can be shown [15] that $E[[\{e_{i+1}\} [e_{i+1}]]$ takes the minimum value, when

$$L_i = A^*A_i G(R_i^* + G* A_i, G)^{-1}$$  \hspace{1cm} (40)$$
CONTROL SCHEME WITH A KALMAN FILTER

Addition of Tuned Mass Dampers and Actuators

Equation of motion
\[ Q + CQ + KQ = HU + S \left( f_s(t) + f_d(t) \right) \]

State equation
\[ \dot{Q}_{st} = A'Q_{st} + C'U + D'F_{st-1} + D'F_{st} \]

State estimator
\[ \dot{Q}_{st} = A'Q_{st} + C'U + D'F_{st-1} + L (Y_t - \hat{Y}_t) \]

Model output
\[ \hat{Y}_t = GQ_t \]

Objective function
\[ J(\dot{Q}, U) = \frac{1}{2} \sum_{t=0}^{\infty} \left( Q_t^T \dot{Q}_t + Q_t^T \dot{U}_t + U_t^T \dot{U}_t \right) \]

Minimization of objective function

Control force expression
\[ U_t = -E_1Q_t - E_2F_{st-1} \]

Kalman Filter as a state estimator

Determination of observer gain matrix L

Fig. 2. Flowchart for the control scheme with a Kalman filter.

Symmetric properties

FEM model

Fig. 3. Dimensions and FEM mesh for the symmetric tower.
satisfying the condition
\[ A_{k+1} = S^* + AA^*A^* T - AA^*G^T \times (R^* + GA^*G^T)^{-1}GA^*A^* T. \] (40)

6.2. Steady-state prediction-type Kalman filter

We assume that the system noise due to the wind and the measurement noises are steady-state noises, so that \( S^* = S^* \) and \( R^* = R^* \), eqn (39) then becomes
\[ L = A^* GG(R^* + G^* A^* G^T)^{-1} \] (41)
satisfying the condition
\[ A = S^* + AA^*A^* T - AA^*A^* T \times (R^* + GA^*G^T)^{-1}GA^*A^* T \] (42)
where \( S^* \) can be obtained from the power spectra of the buffeting force. Comparing eqns (41) and (42) for the determination of observer gain matrix \( L \) and the parameter matrix \( A \) with those in ref. [7], it was found that they are similar except in Ref. [7], \( S^* = I \) and \( R^* = 0 \). Measurement noise was not considered in Ref. [7].

The control algorithm along with the development of system equations can be explained through a flow chart, as shown in Fig. 2.

7. NUMERICAL RESULTS AND COMPARISONS

7.1. Structure description

A partially braced bridge tower, similar to that of Higashi Kobe Bridge tower, has been simulated for the numerical example. This tower has a special feature that its first two in-plane vibration modes are closely spaced. It has been shown [5, 7, 8] that in such structures galloping is always a single mode phenomenon when the structural properties are symmetric. On the other hand, if the structural properties are unsymmetric, either due to the unsymmetrical distribution of mass or non-uniform cross section, then galloping is a multi-mode phenomenon.

Structure with symmetric properties. The dimensions and FEM model of the simulated tower are shown in Fig. 3 and its corresponding structural properties are given in Table 1. In this study only the in-plane vibration of the tower is considered. As the in-plane motion of the tower is not much affected by the presence of cables [3], and also as the vertical motion effect is very little on the lateral motion and rotation, their effects are neglected. Hence each element is modeled with a beam element having two degrees of freedom, namely an in-plane horizontal displacement and a rotation. The first two natural frequencies of this simulated tower are found to be 2.79 and 2.99 rad s\(^{-1}\). As these frequencies are considerably close to each other, they are selected in order to observe the multi-mode galloping behavior of the bridge tower. The corresponding mode shapes are shown in Fig. 4. The eigen modes are normalized by the eigen mode component at the top of the tower at point \( P \). The model damping ratios for the two modes are assumed to be 0.0016 and 0.0014 and these values are available in the literature [5].

Structure with unsymmetric properties. To observe the multi-mode galloping an additional 40 tons load has been added to one leg of the tower so that its mass distribution becomes unsymmetrical, as shown in Fig. 5. The additional load may be considered as a crane mounted on one of the legs of the tower during the

<table>
<thead>
<tr>
<th>Segment number</th>
<th>Mass per unit length (t m(^{-1}))</th>
<th>Moment of inertia (m(^4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16.273</td>
<td>2.45</td>
</tr>
<tr>
<td>2</td>
<td>10.921</td>
<td>2.35</td>
</tr>
<tr>
<td>3</td>
<td>16.840</td>
<td>2.50</td>
</tr>
<tr>
<td>4</td>
<td>13.706</td>
<td>1.808</td>
</tr>
<tr>
<td>5</td>
<td>13.353</td>
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<tr>
<td>6</td>
<td>7.696</td>
<td>4.435</td>
</tr>
<tr>
<td>7</td>
<td>9.812</td>
<td>6.966</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1st mode</th>
<th>2nd mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric properties</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4. Mode shapes of the symmetric tower.
Table 2. Vibration properties of the two cases

<table>
<thead>
<tr>
<th>Vibration properties</th>
<th>Symmetric properties</th>
<th>Unsymmetric properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode 1</td>
<td>Mode 2</td>
</tr>
<tr>
<td>Circular frequency (rad s(^{-1}))</td>
<td>2.79</td>
<td>2.99</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>0.0016</td>
<td>0.0014</td>
</tr>
<tr>
<td>Mass ratio</td>
<td>0.000163</td>
<td>0.000167</td>
</tr>
<tr>
<td>Galloping onset velocity (m s(^{-1}))</td>
<td>16.3</td>
<td>14.8</td>
</tr>
</tbody>
</table>

Table 3. Aerodynamic coefficients for the two cases

<table>
<thead>
<tr>
<th>Aerodynamic coefficients</th>
<th>Symmetric properties</th>
<th>Unsymmetric properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode 1</td>
<td>Mode 2</td>
</tr>
<tr>
<td>( C_{x\cdot 1} )</td>
<td>( 3.08 \times 10^0 )</td>
<td>( 3.09 \times 10^0 )</td>
</tr>
<tr>
<td>( C_{x\cdot 2} )</td>
<td>( 8.06 \times 10^{-1} )</td>
<td>(-1.00 \times 10^{-1})</td>
</tr>
<tr>
<td>( C_{x\cdot 3} )</td>
<td>(-8.80 \times 10^{-1} )</td>
<td>( 8.73 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 4} )</td>
<td>( 7.68 \times 10^{-1} )</td>
<td>( 4.58 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 5} )</td>
<td>( 4.03 \times 10^{-1} )</td>
<td>( 8.37 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 6} )</td>
<td>( 2.58 \times 10^{-1} )</td>
<td>(-6.19 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 7} )</td>
<td>(-5.45 \times 10^{-1} )</td>
<td>( 2.58 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 8} )</td>
<td>( 2.51 \times 10^{-1} )</td>
<td>(-1.44 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 9} )</td>
<td>(-1.27 \times 10^{-1} )</td>
<td>( 2.77 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 10} )</td>
<td>( 2.44 \times 10^{-1} )</td>
<td>( 3.06 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 11} )</td>
<td>( 2.69 \times 10^{-1} )</td>
<td>( 2.71 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 12} )</td>
<td>( 8.96 \times 10^{-2} )</td>
<td>(-3.19 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 13} )</td>
<td>(-2.81 \times 10^{-1} )</td>
<td>( 1.00 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 14} )</td>
<td>( 8.80 \times 10^{-2} )</td>
<td>(-1.53 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 15} )</td>
<td>(-1.35 \times 10^{-1} )</td>
<td>( 9.83 \times 10^{-2} )</td>
</tr>
<tr>
<td>( C_{x\cdot 16} )</td>
<td>( 8.65 \times 10^{-2} )</td>
<td>( 7.40 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 17} )</td>
<td>( 6.51 \times 10^{-13} )</td>
<td>( 9.68 \times 10^{-2} )</td>
</tr>
<tr>
<td>( C_{x\cdot 18} )</td>
<td>( 8.51 \times 10^{-2} )</td>
<td>( 1.62 \times 10^{-1} )</td>
</tr>
<tr>
<td>( C_{x\cdot 19} )</td>
<td>( 1.43 \times 10^{-11} )</td>
<td>( 9.53 \times 10^{-2} )</td>
</tr>
</tbody>
</table>

\* indicates the mode number.

40 Tons Load  Additional Load

Unit: m

Unsymmetric properties

FEM model

Fig. 5. Dimensions and FEM mesh for the unsymmetric tower.
construction operation [5]. The first two in-plane natural frequencies of this modified tower are now 2.66 and 2.99 rad s\(^{-1}\). The mass ratios for the two modes are found to be 1.46 and 1.64 \(\times 10^{-4}\). The coefficients \(C_{\text{bmk}}\) for this tower are also shown in Table 3.

7.2. Galloping phenomenon

Because of the non-availability of the aerodynamic coefficients \(A_4, A_5, A_6,\) and \(A_7\) of this cross section of tower in the literature, and since the tower legs are similar to the 2:3 rectangular section, the aerodynamic coefficients are assumed to be that of a 2:3 rectangular section. These coefficients are obtained by the curve fitting of the coefficient of lifts for a 2:3 rectangular section \[16\] and are shown in Fig. 6. The critical wind velocities \((V_{\text{crit}} = \frac{\nu_{\text{cr}}}{h/C_{\text{d,at}} A_1})\) for the onset of galloping for the first and second modes are calculated to be 16.3 and 14.8 m s\(^{-1}\), respectively (at the lower top 25.5 and 23.3 m s\(^{-1}\), respectively, with \(p = 1/6\)), and for the unsymmetric tower these are found to be 17.3 and 14.8 m s\(^{-1}\), respectively. Galloping behavior for the symmetric and unsymmetric towers are observed under the various initial conditions of excitation, such as

(i) first mode initial excitation,
(ii) second mode initial excitation,
(iii) mixed mode initial excitation.

It has been observed [5, 8] that in all the cases, the symmetric tower converges to single mode galloping, i.e. only one mode vibrates and the other mode remains dormant. On the other hand in the case of an unsymmetric tower, it was found that in all cases multi-mode galloping takes place.

7.3. Galloping control

The unsymmetric tower, in which the multi-mode galloping is observed, is taken as a representative case for galloping control. A turbulent wind condition is assumed. For the gust load, the surface drag coefficient \(K_0\) is assumed to be 0.003, which is taken from the Ref. [12]. Again because of the non-availability of the actual \(C'_b\) and \(C_t\) values for this section of tower in the literature, we have conservatively assumed \(C'_b\) as the slope of the steepest part of Fig. 6, which is approximately 8.0 and \(C_t = 0.4 (C'_b = 4.22\) and \(C_t = 0.27\) for bridge deck taken by Scanlan and Nicholas [10]).

Two dampers are placed at the top of each leg of the tower and are tuned with the first and second modes of the tower. For active control, two actuators are provided with the two TMDs. Also two displacement sensors are provided, one at the top and the other at the mid height of the tower. The TMD mass ratio has been defined as the ratio of the generalized mass of the TMD to that of the generalized modal mass of the structure. The mass ratio for each TMD is assumed to be 0.5\%, whereas the damping in each TMD is assumed to be 5\% of the critical. Because of the practical considerations, the relative displacement of the TMD with respect to the tower cannot exceed \(h/2\). Here we have assumed \(-1.0\ m \leq Y_i \leq 1.0\ m\).

The value of matrix \(S^*\) is obtained from eqn (16). In the actual practice matrix \(R^*\) can be obtained from the covariance of the measurement noise, but here, in this numerical example, \(R^*\) has been determined by the trial and error such as to result in the most optimal value of \(L\).

Figure 7(a) shows the time history response of the uncontrolled vibration of the tower at the location \(P\), as shown in Fig. 3. It can be seen that due to the simultaneous galloping of the two modes, a nearly

![Fig. 7](image-url)

Fig. 7. Time history of uncontrolled and controlled displacement responses with passive and active TMDs and the control force at the top of the tower.
beating effect is produced. In some part of the motion, the two modes reinforce each other and in other parts, they cancel each other. The resultant amplitude of vibration is increasing with the passage of time. Figure 7(b) shows the time history response of the controlled vibration with passive TMD. It can be seen that the addition of a TMD results in the smaller amplitude of vibration. However, the reduction in the amplitude is merely a moderate one, whereas Fig. 7(c) shows the controlled vibration with an active TMD. It is evident that the Active TMD results in the desired control. Figure 7(d) shows the time history response of the control force also located at P. It can be seen that the value of control force (actuator force) decreases with the controlled vibration amplitude showing a linear relationship between the control force and the response. Figure 8 shows the displacement responses of the passive and active TMDs located at P. It can be seen from Fig. 8(b) that when the displacement of the TMD reaches about 1 m with respect to the tower, the motion of the TMD is stopped through a buffer. It is also suggested that this kind of constraint should be included in the desired objective function [eqn (30)] in order to make a mathematically more complete model.

In order to observe the effectiveness of ATMD with the decreasing TMD mass, the performance of both ATMD and Passive TMD are plotted against the TMD mass ratio. For the effective comparison, the symmetric structure is considered. In Fig. 9, the rms responses of the controlled responses normalized by the uncontrolled rms responses at the various TMD mass ratios are furnished (the time interval for the comparison is taken as 60 s). It can be seen from Fig. 9 that with the decreasing mass ratio of TMD, the performance of the passive TMD deteriorates. On the other hand the performance of Active TMD remains unchanged.

8. CONCLUSION

A robust active control algorithm with a dual TMD system has been proposed for the control of structures undergoing galloping. The Kalman filter approach has been adopted as a state estimator. In this system, the covariance of the error vector between the model state vector and the actual state vector is minimized through a filtering process. The observer equations [eqns (41) and (42)] contain both the system noise, which is due to the fluctuating wind velocities, and the measurement noise. The comparison of the observer equations of the present system and that used in the optimal observer system [7,8] clearly reveals the superiority of the present system over the previous one. The previous system completely neglects the buffeting force in the observer equation and only relies on the observer measurement, which might result in the divergence of solution in the very extreme turbulent condition. The present system does not have this disadvantage, as it properly
takes into account the system noise due to the buffeting force along with the measurement noise. In this approach a prior knowledge of wind characteristic results in a better state estimator. Special attention is given to the control of multi-mode galloping problem. Numerical studies are carried out to demonstrate the robustness of the presently developed control algorithm. It is found that with the addition of an active device the inherent damping nature of the TMD is enhanced, which is evident from a quicker suppression of vibration with Active TMD. Also with a smaller mass of damper, almost the same performance in the Active Tuned Mass Damper is achieved, whereas the performance of Passive TMD degrades progressively with the decreasing TMD mass.

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REFERENCES